LQG feedforward regulator design for deterministic and stochastic disturbances

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Abstract

Linear feedforward regulators can be designed by solving a spectral factorization and a linear polynomial equation. A quadratic criterion with penalty on a filtered control signal is then minimized. This constitutes a simple systematic way to design feedforward regulators for non-minimum phase systems. The approach is discussed for disturbances modelled by systems with poles on the stability limit. Regulators can be designed for shape-deterministic disturbances, such as random step and ramp sequences, in the same way as for stochastic disturbances.

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I Introduction

Feedforward regulators utilize measurements of important disturbances. When there is a delay between a measurable disturbance and the controlled output, a feedforward regulator can react on the disturbance before it begins to affect the controlled variable. Sometimes, virtually complete disturbance cancellation may be achieved.

LQG optimization is an attractive framework for studying feedforward design problems. It provides tradeoffs between input energy and disturbance rejection. Control of systems with input delay or non-minimum phase dynamics becomes straightforward. The design of combined feedback-feedforward regulators using LQG optimization based on polynomial equations [5] has received considerable interest recently [4],[9], [11], [13],[14].

Feedforward control is in general complemented with output feedback. Improved robustness, the need to reject unmeasurable disturbances and to stabilize unstable systems are three motives for this. In this note, we will however discuss disturbance feedforward *without* output feedback for scalar discrete time systems. Measurement of controlled variables may sometimes be expensive or impossible. Furthermore, the simplicity of the solution provides many insights into the controller structure. The LQG feedforward regulator also has a close correspondence to other optimal open loop filters, such as servo controllers [7], [11], [15] and deconvolution or input estimation filters [1], [6], [12].

The feedforward filter is calculated via a spectral factorization and a linear polynomial equation. This design procedure has been derived previously for stationary stochastic disturbances [10], [13]. It is here being applied also to shape-deterministic and deterministic signals. This includes random step sequences, ramp sequences and sinusoids. Besides of their frequent occurence in practice, these cases are of some theoretical interest. With disturbance models having poles on the stability limit, they represent the very limit of applicability of LQG design methods.

II The feedforward control problem

Let the plant be described by the following linear discrete time model

$$A(q^{-1})y(t) = B(q^{-1})u(t-k) + D(q^{-1})w(t-d) \tag{1}$$

where the output $y(t)$, input $u(t)$ and measurable disturbance $w(t)$ are all scalar signals. The polynomials A, B and D in the backward shift operator q^{-1} have degrees na, nb and nd. While $A(q^{-1})$ is assumed monic and stable, $B(q^{-1})$ may be unstable. The delays $k > 0$ and $d \geq 0$ may be such that $k > d$; perfect disturbance cancellation is then impossible.

The disturbance $w(t)$ is modelled as a filtered white and zero mean stationary sequence $v(t)$

$$w(t) = \frac{G(q^{-1})}{H(q^{-1})} v(t) \tag{2}$$

with $H(q^{-1}) = H_s(q^{-1})H_u(q^{-1})$. The polynomials $G(q^{-1})$ and $H_s(q^{-1})$ are stable and monic, while $H_u(q^{-1})$ has zeros exclusively on the unit circle. See also Section III below.

The goal is to design a stable and causal time-invariant feedforward regulator

$$u(t) = -\frac{Q(q^{-1})}{P(q^{-1})} w(t) \tag{3}$$

where $P(q^{-1})$ is monic, such that the infinite horizon criterion

$$J = \lim_{N\to\infty} \frac{1}{N} \sum_{t=0}^{N} Ey(t)^2 + \rho E(\tilde{\Delta}(q^{-1})u(t))^2 \tag{4}$$

is minimized. The input penalty $\rho \geq 0$ and the polynomial $\tilde{\Delta}(q^{-1})$ in (4) are chosen by the designer. $\tilde{\Delta}(q^{-1}) = 1 - q^{-1}$ for example represents a differential input penalty.

The solution to one special case of the problem is evident: if maximal disturbance rejection is desired ($\rho = 0$), and if $B(q^{-1})$ is stable and $d \geq k$, the standard feedforward filter

$$u(t) = -q^{-d+k} \frac{D(q^{-1})}{B(q^{-1})} w(t) \tag{5}$$

solves the problem by achieving perfect feedforward control ($y(t) = 0$). A solution to the general problem is presented in Section IV.

III The disturbance model

Let us enumerate the classes of disturbances covered by the model (2)

1. *Stationary stochastic disturbances.* With $H_u = 1$ and $v(t)$ being a white noise with variance Λ_v, we have an ARMA model for $w(t)$.

2. *Drifting stochastic disturbances.* If $w(t)$ e. g. has stationary increments, it is modelled by $H_u = 1 - q^{-1} \triangleq \Delta(q^{-1})$, and a white noise sequence $v(t)$.

3. *Shape-deterministic* or piecewise deterministic signals. Such disturbances have known shape, but unknown magnitude. They occur repeatedly at unknown time instants [2]. They will be modelled by a stationary random spike sequence $v(t)$, such as a Bernoulli-Gaussian sequence [1] ,filtered through $G(q^{-1})/H(q^{-1})$ which generates the known shape. Sequences of random steps are e. g. described by $G(q^{-1}) = H_s(q^{-1}) = 1$, $H_u(q^{-1}) = \Delta(q^{-1})$, and random ramp sequences by $H_u(q^{-1}) = \Delta(q^{-1})^2$.

4. *Deterministic signals* are described by autonomus difference equations $H_u(q^{-1})w(t) = 0$, with nonzero initial values. For multiple sinusoids with frequencies $\{\omega_i\}_1^n$, we have $H_u(e^{j\omega_i}) = 0$. Optimal control of the disturbance types 1, 2 and 3 above results in outputs with finite (and in general nonzero) power. Control of a deterministic disturbance would however result in only one initial transient, with finite energy and zero power measured for $t \in [0, \infty)$. The criterion (4) would be zero in such cases because of the division by $1/N$. To include deterministic signals in our optimization framework, they are treated as shape-deterministic. Their transient phase, which is to be optimized, is formally considered to be repeated. We use the model $H_u(q^{-1})w(t) = v(t)$, where $v(t)$ is a random spike sequence.

IV The optimal filter

In order to obtain a convenient notation, substitute z for q^{-1} and define , for any polynomial $P(z)$, $P_* = P(z^{-1})$ and $\overline{P} = z^{np}P_*$. Note that the zeros of $\overline{P}$ are reflected in the unit circle, compared to those of P. The polynomial arguments will in general be omitted in the following.

Introduce the spectral factorization

$$r\beta\beta_* = BB_* + \rho A\tilde{\Delta}\tilde{\Delta}_* A_* \tag{6}$$

where r is a positive scalar, and β is a stable monic polynomial in z with degree $n\beta$. To assure that a stable β exists, we require B to have no zero on the unit circle when $\rho = 0$, while B and $\tilde{\Delta}$ have no common factors with zeros on the unit circle when $\rho > 0$.

Theorem 1

Assume that H_u is a factor of either $\tilde{\Delta}$ or D, and that a stable spectral factor β in (6) exists. The feedforward regulator (3) then attains the global minimum value of (4), under the constraint of stability, if

[1] A Bernoulli-Gaussian sequence is given by $v(t) = r(t)s(t)$ where $s(t)$ is a Bernoulli sequence such that $s(t) = 1$ w. p. λ and $s(t) = 0$ w. p. $1 - \lambda$. $r(t)$ is a zero mean Gaussian sequence with variance σ^2 independent of t [6]. It is then straightforward to show that $v(t)$ is a stationary white sequence with zero mean and variance $\Lambda_v = \sigma^2\lambda$.

$$P = \beta G \tag{7}$$

and if Q, together with a polynomial L, is the unique solution to the polynomial equation

$$z^{-d+k} B D_* G_* = r\beta Q_* + A_* H_* z L \tag{8}$$

The minimal criterion value is finite, and is given by

$$J_{min} = \frac{\Lambda_v}{2\pi i} \oint_{|z|=1} \frac{LL_*}{r\beta\beta_*} \frac{dz}{z} + \frac{\rho\Lambda_v}{2\pi i} \oint_{|z|=1} \frac{GG_* DD_* \tilde{\Delta}\tilde{\Delta}_*}{r\beta\beta_* HH_*} \frac{dz}{z} \tag{9}$$

□

Proof: See the Appendix.

Remarks:

1. Since β (stable) and $z^{na} A_* z^{nh} H_* = \overline{AH}$ (unstable) cannot have common factors, the linear equation (8) will be solvable. The degrees of $Q_*(z^{-1})$ and $L(z)$ are chosen so that the maximal occuring power in z^{-1} and z, respectively, are covered [2]:

$$\begin{aligned} nQ &= \max\{na + nh - 1, nd + ng + d - k\} \\ nL &= \max\{n\beta, nb - d + k\} - 1 \end{aligned} \tag{10}$$

   Note that the solution of only one Diophantine equation, namely (8), is sufficient for optimizing the feedforward filter. LQG optimization of feedback regulators in general requires the solution of two coupled Diophantine equations [5].

2. Minimum variance feedforward controllers ($\rho = 0$) have a straightforward interpretation: with $P = \beta G$, such regulators have poles in the locations of stable system zeros, and in the *inverse points with respect to the unit circle* of unstable zeros. In addition, the (stable) disturbance model zeros are cancelled. (Cancellation of stable zeros of B on the negative real axis may, however, result in an oscillative input, and hidden inter-sample oscillations in the output. Use of a small input penalty solves this problem.) When $d \geq k$ and B is stable, a minimum variance regulator of course reduces to the perfect feedforward filter (5).

[2]These choices are unique: with higher degrees, the superfluous coefficients would be zero. With lower degrees, no solution can be found. If (10) assigns the degree -1, the corresponding polynomial should be set to zero. (The interpretation of $nQ = -1$ is that no regulator could improve the criterion value.)

3. Assume random step disturbances. Thus, $v(t)$ is a random spike sequence, $H_u = 1 - q^{-1} = \Delta$ and $\tilde{\Delta} = \tilde{\Delta}_1 \Delta$ for some $\tilde{\Delta}_1(q^{-1})$. The regulator then has correct static gain $D(1)/B(1)$. Since $D(1) = D_*(1)$ for any polynomial D, the spectral factorization (6) gives $r\beta(1)\beta(1) = B(1)B(1) + 0$. From (7) and (8),

$$\begin{aligned} P(1) &= \beta(1)G(1) \\ B(1)D(1)G(1) &= r\beta(1)Q(1) + A(1)H(1)L(1) \end{aligned}$$

But $H(1) = 0$ since $H = H_s\Delta$. Thus,

$$\frac{Q(1)}{P(1)} = \frac{B(1)D(1)G(1)}{r\beta(1)\beta(1)G(1)} = \frac{D(1)}{B(1)} \tag{11}$$

Consequently, the output goes to zero asymptotically after each step disturbance. This is true for any input penalty ρ. When $\rho = 0$, the squared disturbance step response will be minimized.

4. That H_u must be a factor of either $\tilde{\Delta}$ or D is evident from (9). Consider drifting stochastic disturbances: if $H_u = \Delta$ is a factor of D, the output of the uncontrolled system will not be drifting. Otherwise, a drifting control signal is needed. To keep the criterion finite, Δ must then be a factor of $\tilde{\Delta}$. Note that unstable disturbance model factors H_u will *not* be factors of P. This is in contrast to LQG output feedback or combined feedback-feedforward control, where H_u is a regulator factor [9],[11]. The internal modelling principle [3] does not apply to feedforward controllers.

5. The main drawback with feedforward control, where the controlled variable is not (or perhaps cannot be) measured, is that it requires precise model knowledge. For drifting stochastic disturbances, a residual drift would for example in practice be present in the controlled output. The relation (11) never holds exactly. The requirement for perfect cancellation can also cause theoretical difficulties in cases when $H_u \neq 1$. At points arbitrarily close to the optimal one in regulator parameter space, the criterion then becomes undefined. Early Wiener filtering approaches to controller design [8] relied on differentiation of J w. r. t. regulator parameters. Not surprisingly, they encoutered difficulties with these problems.

6. For sinusoid disturbances, the system and regulator together constitute a notch filter. The feedforward regulator will have correct gain and phase shift at the sinusoid frequencies ω_i to cancel them asymptotically. This becomes evident by repeating the calculation of (11) for $Q(e^{j\omega_i})/P(e^{j\omega_i})$. The choices of ρ and $\tilde{\Delta}_1(q^{-1})$ in $\tilde{\Delta} = \tilde{\Delta}_1 H_u$ only affect the transients when the sinusoids change magnitude or phase.

If the measurement $w(t)$ is influenced by the input $u(t-n), n>0$, this effect could be subtracted internally, inside the regulator [11]. If the feedforward signal is corrupted by a coloured measurement noise

$$w_m(t) = w(t) + \frac{F(q^{-1})}{K(q^{-1})}m(t) \tag{12}$$

an additional spectral factorization has to be introduced:

$$s\gamma\gamma_* = GG_*KK_* + \frac{\Lambda_m}{\Lambda_v}FF_*HH_* \tag{13}$$

where Λ_m is the variance of the white noise $m(t)$, and γ is stable and monic.

With the same approach as in the Appendix, it is then straightforward to show that the criterion (4) is minimized by the feedforward regulator

$$u(t) = -\frac{Q(q^{-1})K(q^{-1})}{\beta(q^{-1})\gamma(q^{-1})}w_m(t) \tag{14}$$

where Q, together with L, is the solution to

$$z^{-d+k}BD_*G_*GK = r\beta s\gamma Q_* + A_*H_*zL$$

Note that the variance ratio Λ_m/Λ_v affects the design, via (13). A high measurement noise level will tend to reduce the filter gain. When $w_m(t)$ is stationary, the regulator (14) can be interpreted as the inverse of an innovations model of $w_m(t)$ (a whitening filter) KH/γ, in cascade with an additional filter $-Q/\beta H$.

V An example

Consider the non-minimum phase system

$$(1-1.4q^{-1}+0.65q^{-2})y(t) = (0.1+0.2q^{-1})u(t-3) + (0.4-0.6q^{-1})w(t-2) \tag{15}$$

where $w(t)$ is a random sequence of step disturbances, modelled by $G=1$, $H=1-q^{-1}$. With $\tilde{\Delta}=1-q^{-1}$, the spectral factorization (6) is given by

$$\begin{aligned} r\beta(z)\beta_*(z^{-1}) &= (0.1+0.2z)(0.1+0.2z^{-1})+ \\ &\quad + \rho(1-1.4z+0.65z^2)(1-z)(1-z^{-1})(1-1.4z^{-1}+0.65z^{-2}) \end{aligned}$$

Reliable algorithms for spectral factorization are found in e. g. [5]. Explicite solutions exist for $n\beta \le 2$ [9]. Consider the feedforward filter corresponding to $\rho = 0.1$. This gives

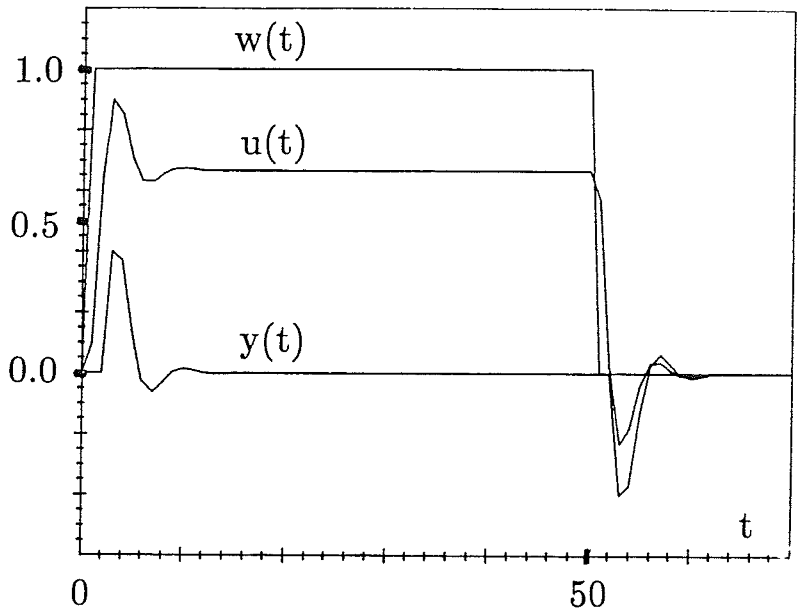

Figure 1: Measured disturbance $w(t)$, control signal $u(t)$ and the output $y(t)$. Feedforward control designed with an input penalty $\rho = 0.1$ is applied to the system (15).

$$r = 0.4791 \quad , \quad \beta(z) = 1 - 1.0429z + 0.612z^2 - 0.1357z^3$$

The polynomials Q and L both have degree 2 according to (10). They solve the polynomial equation (8)

$$z^{-2+3}(0.1 + 0.2z)(0.4 - 0.6z^{-1}) = 0.4791\beta(z)(Q_0 + Q_1z^{-1} + Q_2z^{-2}) +$$
$$+ (1 - 1.4z^{-1} + 0.65z^{-2})(1 - z^{-1})z(\ell_0 + \ell_1 z + \ell_2 z^2)$$

which can be rewritten as a linear system of simultaneous equations. The solution is

$$Q_*(z^{-1}) = -0.09568 - 0.4456z^{-1} + 0.2523z^{-2}$$

$$L(z) = 0.1860 + 0.06417z - 0.006219z^2$$

The control performance of the resulting feedforward regulator

$$u(t) = \frac{0.09568 + 0.4456q^{-1} - 0.2523q^{-2}}{1 - 1.0429q^{-1} + 0.612q^{-2} - 0.1357q^{-3}} w(t) \tag{16}$$

is shown in Figure 1 for a unit step disturbance.

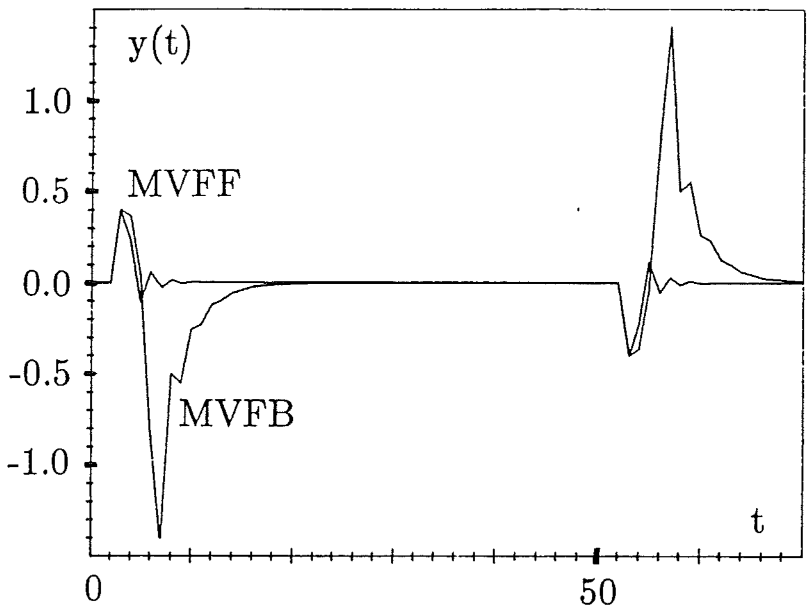

Figure 2: Unit disturbance step responses when minimum variance feedforward control (MVFF) and integrating LQG output feedback control with no input penalty (MVFB) is applied. These regulators minimize the squared disturbance step response.

For comparision, Figure 2 shows the result of minimum variance feedforward control ($\rho = 0$). It also shows the result of LQG *output feedback* control with integration [9], [11], also for $\rho = 0$:

$$\Delta u(t) = \frac{-10.606 + 15.95q^{-1} - 7.0147q^{-2}}{1 + 2.234q^{-1} + 2.9786q^{-2} + 2.1584q^{-3}} y(t) \tag{17}$$

This corresponds to pole placement in $\overline{BD} = (1 + 0.5q^{-1})(1 - 0.666q^{-1})$.

The disturbance transient with output feedback (17),(MVFB), is much larger than with feedforward control, because of the disturbance delay $d = 2$. Note that minimum variance feedforward (MVFF) cannot improve the output transient significantly, compared to the use of $\rho = 0.1$,Figure 1. Compared to the regulator (16), both the minimum variance controllers generate much larger control signal variations (not shown).

APPENDIX

First, we show that (7),(8) imply a criterion value given by (9). Secondly, it is shown that (9) is the minimal value.

The use of Parsevals relation, $P = \beta G$ and the spectral factorization (6) in the expression (4) gives

$$J = \frac{\Lambda_v}{2\pi j}\oint_{|z|=1} \frac{(z^d DP - z^k BQ)}{PA}\frac{G}{H}\frac{(z^{-d}D_*P_* - z^{-k}B_*Q_*)}{P_*A_*}\frac{G_*}{H_*}\frac{dz}{z} +$$

$$+\frac{\Lambda_v}{2\pi j}\oint_{|z|=1}\rho\frac{\tilde{\Delta}QG\tilde{\Delta}_*Q_*G_*}{PHP_*H_*}\frac{dz}{z}$$

$$= \frac{\Lambda_v}{2\pi j}\oint \frac{(DD_*GG_*\beta\beta_* - z^{d-k}DG\beta B_*Q_* - z^{k-d}BD_*G_*\beta_*Q + r\beta\beta_*QQ_*)GG_*}{\beta GAH\beta_*G_*A_*H_*}\frac{dz}{z} \tag{18}$$

By using (6),

$$\beta\beta_* = \frac{BB_*}{r} + \frac{\rho}{r}A\tilde{\Delta}\tilde{\Delta}_*A_*$$

and the relation

$$DD_*GG_*\frac{BB_*}{r} - z^{d-k}B_*DG\beta Q_* - z^{k-d}BD_*G_*\beta_*Q + r\beta\beta_*QQ_* =$$

$$= [z^{-d+k}B\frac{D_*G_*}{\sqrt{r}} - \sqrt{r}\beta Q_*][z^{d-k}B_*\frac{DG}{\sqrt{r}} - \sqrt{r}\beta_*Q] =$$

$$= \frac{1}{r}AHL_*A_*H_*L$$

which follows from (8), equation (18) can be expressed as

$$J = \frac{\Lambda_v}{2\pi j}\oint \frac{DD_*GG_*\frac{\rho}{r}A\tilde{\Delta}\tilde{\Delta}_*A_* + \frac{1}{r}AHL_*A_*H_*L}{\beta AH\beta_*A_*H_*}\frac{dz}{z}$$

This expression equals (9). Note that the integrand has no poles on the unit circle, since β is assumed stable, and H_u is cancelled by $\tilde{\Delta}$ or D. Furthermore, $L(z)$ has finite coefficients since (8) is solvable for all $\rho < \infty$. Thus, the criterion is finite. The signals $y(t)$ and $\tilde{\Delta}u(t)$ have finite variance.

Next, we show that (9) is the minimal value. Let an arbitrary linear feedforward control law be written as

$$u(t) = -\frac{Q}{P}w(t) + n(t) \tag{19}$$

where Q/P is calculated according to Theorem 1, and $n(t)$ is an arbitrary additional feedforward control action, computed from a linear combination of measurements $w(t)$ up to time t. It will be shown that it is optimal to choose $n(t) = 0$.

Control of (1), (2) by (19) results in the criterion value

$$J = J_1 + 2J_2 + J_3 \tag{20}$$

where J_1 is given by (9) (J for $n(t) = 0$) and

$$J_2 = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} E\left(q^{-k}\frac{B}{A}n(t)\right)\left(\frac{(q^{-d}DP - q^{-k}BQ)G}{PAH}v(t)\right) -$$

$$-\rho E\tilde{\Delta}n(t)\left(\frac{\tilde{\Delta}QG}{PH}v(t)\right)$$

$$J_3 = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} E\left(\frac{B}{A}n(t)\right)^2 + \rho E(\tilde{\Delta}n(t))^2 \geq 0$$

If $n(t)$ were nonstationary, the ensemble means in J_2 and J_3 would change with time, and the criterion could be undefined. Assume $n(t)$ to be stationary. It can then be expressed as

$$n(t) = \frac{T(q^{-1})}{N(q^{-1})}w(t) = \frac{T(q^{-1})G(q^{-1})}{N(q^{-1})H(q^{-1})}v(t) \tag{21}$$

where N is stable and $T = T_1H_u$ to assure stationarity.

Using (21) and $P = \beta G$, the middle term of (20) can be expressed as

$$2J_2 = \frac{\Lambda_v}{\pi j}\oint_{|z|=1} \frac{TG}{NH}\frac{[z^kB(z^{-d}D_*G_*\beta_* - z^{-k}B_*Q_*) - \rho AA_*\tilde{\Delta}\tilde{\Delta}_*Q_*]}{A\beta_*A_*H_*}\frac{dz}{z}$$

The use of first (6) and then (8) reduces $2J_2$ to

$$2J_2 = \frac{\Lambda_v}{\pi j}\oint_{|z|=1} \frac{TGz\beta_*A_*H_*L}{NHA\beta_*A_*H_*}\frac{dz}{z} = \frac{\Lambda_v}{\pi j}\oint_{|z|=1} \frac{T_1GL}{NH_sA}dz = 0 \tag{22}$$

Since $N(z)$, $H_s(z)$ and $A(z)$ are stable, they have all their zeros outside the unit circle. The integral (22) vanishes because poles inside the integration path are absent.

Thus, $J = J_1 + J_3(n(t))$, where J_1 is independent of $n(t)$ and J_3 is nonnegative. Consequently, J_1 given by (9) is the minimal criterion value, and it is attained by using control according to Theorem 1, i. e. $n(t) = 0$.

□

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