

Sensing Throughput Optimization in Fading Cognitive Multiple Access Channels With Energy Harvesting Secondary Transmitters

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Abstract—The paper investigates the problem of maximizing expected sum throughput in a fading multiple access cognitive radio network when secondary user (SU) transmitters have energy harvesting capability, and perform cooperative spectrum sensing. We formulate the problem as maximization of sum-capacity of the cognitive multiple access network over a finite time horizon subject to a time averaged interference constraint at the primary user (PU) and almost sure energy causality constraints at the SUs. The problem is a mixed integer non-linear program with respect to two decision variables namely spectrum access decision and spectrum sensing decision, and the continuous variables sensing time and transmission power. In general, this problem is known to be NP hard. For optimization over these two decision variables, we use an exhaustive search policy when the length of the time horizon is small, and a heuristic policy for longer horizons. For given values of the decision variables, the problem simplifies into a joint optimization on SU transmission power and sensing time, which is non-convex in nature. We solve the resulting optimization problem as an alternating convex optimization problem for both non-causal and causal channel state information and harvested energy information patterns at the SU base station (SBS) or fusion center (FC). We present an analytic solution for the non-causal scenario with infinite battery capacity for a general finite horizon problem. We formulate the problem with causal information and finite battery capacity as a stochastic control problem and solve it using the technique of dynamic programming. Numerical results are presented to illustrate the performance of the various algorithms.

I. INTRODUCTION

Spectrum scarcity is a significant issue in modern wireless networks. This is due to the legacy of static allocation policy of the radio spectrum, which prohibits unlicensed users to exploit licensed spectrum even when it is idle. As a solution, the paradigm of cognitive radio (CR) [1] has been proposed. In the *interweave* paradigm of CR, based on the concept of dynamic spectrum allocation strategy, unlicensed SUs can access the PU licensed spectrum when the PU is idle. The SUs have to vacate the licensed spectrum as soon as the PU becomes active. To achieve this, the SUs sense the spectrum to check whether the PU is active or not. The decision about spectrum sensing is then taken in a cooperative manner by sending all local spectrum sensing decision to an FC which makes an overall decision regarding spectrum access.

The issue of energy efficiency is also a very important aspect of wireless transmission. While traditionally mobile devices have relied on rechargeable batteries, in many situations, periodic battery replacement of the wireless nodes is not a feasible option in practice, such as in sensor networks.

Thus energy harvesting from natural sources like wind or solar power is a viable and cost-effective solution for replenishing energy. Recently, there has been significant research in the domain of energy harvesting in wireless environment [2]. To analyze the performance of energy harvesting cognitive multiple access networks, throughput is generally used as the performance measure. For such a network, sum capacity with different wireless channel models [3] has been investigated. As spectrum sensing is one of the key tasks in a CR network, sensing-throughput analysis has also been studied rigorously in [4]. Capacity analysis of wireless system with energy harvesting capability has been studied in [5] as well. Incorporating the capability of energy harvesting in CR network has also been investigated in [6], [7]. The research in this specific field focuses on both aspects of mitigating spectrum scarcity and efficient energy usage. Recently in [8], the authors have investigated achievable throughput optimization in energy harvesting CR networks where the optimization is over sensing time and sensing threshold.

As opposed to previous work in this area, we investigate a sensing-throughput optimization problem of a block fading multiple access CR network with a single PU, where all CR nodes (or SUs) are equipped with energy harvesting capability, and perform cooperative spectrum sensing. In this work, we investigate the tradeoff between sensing time and sum capacity of the CR multiple access channel with respect to *transmission power* and *sensing time*, keeping the sensing threshold fixed. Since spectrum sensing also consumes energy, due to the unreliable nature of harvested energy patterns, the CR nodes must make an initial decision as to whether they would perform spectrum sensing or not. All participating CR nodes' sensing decisions are combined at the SBS (also performing as the FC) to arrive at a final spectrum access decision which is broadcast to all CR nodes. If the PU is deemed to be absent, the participating CR nodes use the remaining time in each fading block for information transmission with a suitable transmission power. The problem we investigate is mixed integer nonlinear programming (MINLP) problem with respect to the individual spectrum sensing (whether or not to participate) and the overall spectrum access decisions, sensing time and transmission power of each user. We consider both non-causal and causal channel state information (CSI) and harvested energy information (or information on the battery level at each user) at the FC/SBS, for optimizing the CR multiple access channel sum capacity over a finite time horizon, where each user's energy consumption in each fading block cannot exceed their battery level at the beginning of

the block (energy causality constraint), a peak transmission power constraint and an average interference constraint at the PU receiver.

Our contributions are listed as follows:

- The problem of sum-capacity maximization in a multiple access cognitive network environment is investigated with energy harvesting and both infinite/finite battery settings. We first derive an analytic closed form solution for the general horizon problem for the case of non-causal CSI. Since the problem is individually convex with respect to the sensing time or the transmission power, but not jointly, we implement an iterative convex optimization scheme which is provably convergent to a locally optimal solution.
- For short horizon lengths, we employ an exhaustive search to find the optimal values of the Boolean variables, namely the spectrums sensing and access decisions. For longer horizons, motivated by the NP hard nature of the problem, we propose a heuristic algorithm to determine these variables in a suboptimal manner.
- The problem involving the more realistic scenario of finite battery and causal CSI and harvested energy is solved using the dynamic programming (DP) algorithm with discretized levels of power and sensing times.
- We present illustrative numerical results to demonstrate the comparative performance of the various algorithms proposed in this work.

The rest of the paper is organized as follows. In section II we discuss the system model. In section III we describe the optimization problem. In section IV we propose the closed form solution of the problem for the infinite battery and non-causal CSI scenario. In section V we discuss the policy of causal finite battery scenario using the DP algorithm. Section VI contains simulation results, followed by concluding remarks in section VII.

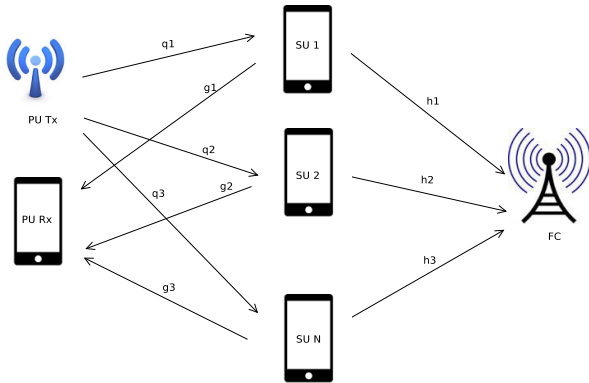


Figure 1: System model for Cognitive MAC.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We study the system model as depicted in Fig.1. The system under consideration has N number of SUs communicating to the SBS/FC. In the adopted cooperative spectrum sensing model each SU decides whether to participate or not in sensing the PU spectrum (due to the random nature of harvested energy), and sends its individual spectrum sensing decision (if participating) to the FC, which makes the overall decision and broadcasts it to all the SUs.

We assume that time is slotted, where each slot represents a block where the all fading channels remain invariant and change from one block to another in a statistically independent manner. All relevant parameters, random variables and optimization variables used in this paper are described in Table I. Each time slot of T time units is utilized by the SUs for the spectrum sensing and data transmission. In the k^{th} time slot SUs spend τ_k time units for spectrum sensing and remainder of the slot ($T - \tau_k$) time units for data transmission, provided the spectrum sensing result indicates PU spectrum vacancy. In the k^{th} time slot, i^{th} SU first makes a decision to perform spectrum sensing or to be idle on the basis of the individual decision to sense $a_{i,k}$, $1 \leq i \leq N$, $1 \leq k \leq M$, where M is the length of time horizon over which the system performance is optimized, and $a_{i,k} \in \{0, 1\}$, 1 (or 0) representing the decision to perform (or not to perform) spectrum sensing. The decision to sense $a_{i,k}$ is obtained by the individual SUs by the following rule [9] $a_{i,k} = 1$, if $B_{i,k} \geq p_s \tau_k$ and 0 otherwise, where p_s is the sensing power and $B_{i,k}$ is the battery state in the i^{th} SU at the beginning of the k^{th} slot. With a battery of finite capacity B_{max} , $B_{i,k}$ can be expressed as:

$$B_{i,k+1} = \min \{B_{max}, B_{i,k} - E_{i,k} + H_{i,k}\} \quad (1)$$

where $H_{i,k}$ is the energy harvested by the i^{th} SU in the k^{th} time slot. In (1), $E_{i,k}$ is the amount of energy used by i^{th} SU at k^{th} time slot, which can be expressed as

$$E_{i,k} = a_{i,k}(p_s \tau_k + p_{i,k}(T - \tau_k)(1 - \theta_k)) \quad (2)$$

where θ_k is the overall spectrum access decision indicating PU is present if $\theta_k = 1$ (details in the next subsection). One can also express $B_{i,k}$ as

$$B_{i,k} = \min \left\{ B_{max}, B_i - \sum_{r=1}^k E_{i,r} + \sum_{r=1}^{k-1} H_{i,r} \right\} \quad (3)$$

where B_i is the initial battery level of i^{th} SU. Note that in the infinite battery scenario $B_{i,k}$ from (3) simplifies to $B_{i,k} = B_i - \sum_{r=1}^k E_{i,r} + \sum_{r=1}^{k-1} H_{i,r}$.

A. Spectrum Sensing Model

If an SU decides to sense, it collects the samples of the received signal from PU by dividing sensing time interval τ_k to a number of mini-slots, where the length of the mini-slots is a constant and pre-decided. The PU spectrum availability is decided by the following received signal model under hypothesis \mathcal{H}_0 (PU absent) and \mathcal{H}_1 (PU present).

$$\begin{aligned} \mathcal{H}_0 : y_{i,k,m} &= n_{i,k,m} \\ \mathcal{H}_1 : y_{i,k,m} &= q_i x_k + n_{i,k,m} \end{aligned} \quad (4)$$

where x_k denotes the PU's transmitted signal for the k^{th} time slot, where it is assumed to be real valued and distributed as $x_k \sim \mathcal{N}(0, \sigma_x^2)$. The parameter q_i is the CSI between PU transmitter and i^{th} SU sensing device, which is assumed to be known at the SU throughout the spectrum sensing process. The parameters $y_{i,k,m}$ and $n_{i,k,m}$ are the real valued received signal and noise components respectively for the i^{th} SU, in k^{th} time slot and m^{th} mini-slot. The noise is distributed as $n_{i,k,m} \sim \mathcal{N}(0, \sigma_n^2)$. We adopt the energy detection policy of [10] for each SU. The local spectrum sensing decision at the i^{th} SU in k^{th} horizon is defined as $\theta_{i,k} = \mathcal{I} \left\{ \frac{1}{S} \sum_{m=1}^S y_{i,k,m}^2 \geq \gamma \right\}$, where \mathcal{I} is the indicator function, S is the number of mini-slots in a particular time slot

and γ is the detection energy threshold. These local decisions are sent to the FC by error-free control channels and combined using the OR logic fusion. In this scenario, probability of false alarm is $P_{fa} = Pr\{\theta_k = 1 | \mathcal{H}_0\}$.

We make the following implicit assumptions regarding the system model under consideration. We consider the channel power gains between the SU transmitters and FC as well as SU transmitters and PU receiver being distributed as exponential random variables with unity mean, without loss of generality. It is also assumed that SUs have the ability of mitigating the interference caused by PU. This assumption simplifies the expression of sum capacity with respect to the scenario which accounts for PU interference as a random parameter, although the proposed algorithms in this paper can be extended to the case where PU interference can be explicitly considered. We also model the primary user activity as a stationary random process, which is assumed to occur with probability κ (of PU being present) within each time slot.

Table I: System Parameters

τ_k	Time taken to perform the spectrum sensing in the k^{th} slot.
$p_{i,k}$	Transmission power for i^{th} SU in the k^{th} time slot.
κ	PU activity probability.
$h_{i,k}$	The CSI between i^{th} SU Transmitter and FC in the k^{th} time slot.
$g_{i,k}$	The CSI between i^{th} SU Transmitter and PU Receiver in the k^{th} time slot.
q_i	The CSI between PU Transmitter and i^{th} SU in the k^{th} time slot.
p_s	Power allocated to sensing.
P_{max}	The peak power limit on $p_{i,k}$.
B_i	The initial battery state for i^{th} user.
$H_{i,k}$	Energy harvested for i^{th} SU for the k^{th} time slot.
$a_{i,k}$	Spectrum access decision variable for the i^{th} SU in k^{th} time slot.
θ_k	Spectrum sensing decision for the k^{th} horizon.
Q_{avg}	The average interference limit to the PU.

B. Sum Capacity Maximization

For a finite horizon of length M , the sensing-throughput optimization problem for a CR multiple access channel with an average interference constraint at the PU receiver and energy causality constraints at each SU node can be formulated as (with $\forall i, 1 \leq i \leq N, \forall k, 1 \leq k \leq M$)

$$\max_{a_{i,k}, \theta_k, p_{i,k}, \tau_k} \mathbb{E} \left\{ \sum_{k=1}^M \frac{T - \tau_k}{MT} \log_2 \left(1 + \sum_{i=1}^N p_{i,k} h_{i,k} a_{i,k} (1 - \theta_k) \right) \right\} \quad (5)$$

$$\text{s.t. } \frac{1}{M} \sum_{k=1}^M \mathbb{E} \left\{ \frac{T - \tau_k}{T} \sum_{i=1}^N p_{i,k} g_{i,k} a_{i,k} (1 - \theta_k) \right\} \leq Q_{avg} \quad (6)$$

$$0 \leq p_{i,k} \leq P_{max}; \quad (7)$$

$$0 \leq \tau_k \leq T; \quad (8)$$

$$E_{i,k} \leq B_{i,k} \quad \text{a.s.} \quad (9)$$

where the last constraint can be replaced by $\sum_{r=1}^k a_{i,r} (p_s \tau_r + p_{i,r} (T - \tau_r) (1 - \theta_r)) \leq B_i + \sum_{r=1}^{k-1} H_{i,r}$ in the case of infinite battery capacity, and **a.s.** stands for *almost surely*. Note also the peak transmission power constraint on each SU, motivated by practical scenarios.

Remark It is important to note that the average interference term in the constraint (6) is normalized by the primary activity factor κ , where $Q_{avg} = Q/\kappa$, Q being the actual interference limit, since no interference is caused when the primary is not active. Note also that although the optimization variables indicate $a_{i,k}, \theta_k, \tau_k$ separately, they are interdependent due to the dependence of $a_{i,k}$ and τ_k , and the dependence of the decision variables $\theta_{i,k}$ on τ_k .

The above mentioned optimization problem is a MINLP problem with respect to the allocated transmission power $p_{i,k}$, the sensing time τ_k , decision to sense $a_{i,k}$ and spectrum sensing result θ_k . In principle, one can employ computationally intensive global optimization methods (e.g. see [11]) to solve such NP hard problems. In this paper, for short horizon lengths, we employ an exhaustive search policy for determining the integer variables $a_{i,k}, \theta_k$. For longer horizons, we will employ a heuristic policy to be described in the next subsection. For the exhaustive search policy, the optimization is performed at the SBS, and all relevant decision variables are exchanged between the SUs and SBS via control channels.

It should be noted that for a fixed choice of $a_{i,k}$ and θ_k , the problem becomes an optimization over $p_{i,k}$ and τ_k , which is a non-convex problem in general jointly in $p_{i,k}$ and τ_k . However, the problem becomes convex if we fix any one of the variables τ_k or $p_{i,k}, \forall i$, which can be solved by convex optimization algorithms. To solve this problem we take the approach of alternating convex optimization (see e.g. [12]), where one alternates between optimizing over τ_k with fixed $p_{i,k}$ and vice versa until the algorithm converges to a local minimum.

C. Heuristic Policy for $a_{i,k}$ and θ_k

The exponential complexity of an exhaustive search algorithm for optimizing over $a_{i,k}$ and θ_k can quickly explode for large values of M and N . For this scenario, we propose a heuristic suboptimal policy as described below. This heuristic policy is based on the idea that limiting the maximum value of P_{fa} on individual SUs would in turn impose a lower bound τ_l on *sensing time* for each horizon. If P_{fa} is bounded by the constraint $P_{fa} \leq \alpha$, for some $\alpha > 0$, then we can have a corresponding lower bound for τ_k as $\tau_l \leq \tau_k$,

which can be obtained by [4] $\tau_l = \frac{1}{f_s} \left\{ \frac{Q^{-1}(\alpha)}{\frac{\sigma_n^2}{\sigma_s^2} - 1} \right\}^2$.

$Q^{-1}(\cdot)$ is the inverse complementary distribution function of the standard Gaussian random variable. Then $a_{i,k} = 1$ if $B_{i,k} > p_s \tau_l$ and vice versa. $\theta_{i,k}$ is determined on the basis of the particular τ_k resulting from the alternating convex optimization method, and the final decision θ_k is obtained by combining the individual $\theta_{i,k}$ at the FC using the OR fusion rule.

III. NON-CAUSAL OPTIMIZATION WITH INFINITE BATTERY

In this section we analyze the problem first in the context of non-causal CSI ($g_{i,k}, h_{i,k}$) and harvested energy ($H_{i,k}$) scenario with the assumption of infinite battery, for fixed values of the integer variables $a_{i,k}$ and θ_k (obtained either via exhaustive search or the heuristic policy). We incorporate the aforementioned alternating convex optimization scheme. For the general M -horizon problem, we obtain a closed form solution for the optimization problem involving $p_{i,k}$, when τ_k is fixed. We also show that the optimization involving τ_k (when $p_{i,k}$ is fixed) is a linear programming problem, which can be solved by any established LP solver.

A. Optimal Power Allocation Policy

For a fixed τ_k , the optimization problem for *transmit power* is given below:

$$\begin{aligned} \max_{p_{i,k}} \quad & \sum_{k=1}^M \frac{T - \tau_k}{MT \log 2} \ln\left(1 + \sum_{i=1}^N p_{i,k} h_{i,k} a_{i,k} (1 - \theta_k)\right) \\ \text{s.t.} \quad & \frac{1}{M} \sum_{k=1}^M \frac{T - \tau_k}{T} \sum_{i=1}^N p_{i,k} g_{i,k} a_{i,k} (1 - \theta_k) \leq Q_{avg} \\ & 0 \leq p_{i,k} \leq P_{max}; 1 \leq i \leq N, 1 \leq k \leq M \end{aligned} \quad (10)$$

$$\sum_{r=1}^k a_{i,r} (p_s \tau_r + p_{i,r} (T - \tau_r) (1 - \theta_r)) \leq B_i + \sum_{r=1}^{k-1} H_{i,r} \quad (11)$$

Remark: For the above problem the expectations with respect to (5) and (6) are removed from the objective function because the random variables corresponding to the channel gains and energy arrival process are non-causally known.

The Lagrangian of the above mentioned convex optimization problem is given by

$$\begin{aligned} \mathcal{L}(\{p_{i,k}\}, \lambda, \delta_{i,k}, \eta_{i,k}, \mu_{i,k}) = & \sum_{k=1}^M \frac{T - \tau_k}{MT \log 2} \times \ln\left(1 + \sum_{i=1}^N p_{i,k} h_{i,k} a_{i,k} (1 - \theta_k)\right) \\ & - \lambda \left(\frac{1}{M} \sum_{k=1}^M \frac{T - \tau_k}{T} \sum_{i=1}^N p_{i,k} g_{i,k} a_{i,k} (1 - \theta_k) - Q_{avg} \right) \\ & + \sum_{k=1}^M \sum_{i=1}^N \eta_{i,k} p_{i,k} - \sum_{k=1}^M \sum_{i=1}^N \delta_{i,k} (p_{i,k} - P_{max}) \\ & - \sum_{k=1}^M \sum_{i=1}^N \mu_{i,k} \left(\sum_{r=1}^k a_{i,r} (p_s \tau_r + p_{i,r} (T - \tau_r) (1 - \theta_r)) \right. \\ & \quad \left. - B_i - \sum_{r=1}^{k-1} H_{i,r} \right) \end{aligned} \quad (13)$$

where $\lambda, \eta_{i,k}, \delta_{i,k}$ and $\mu_{i,k}$ are the non-negative dual variables associated with the average interference constraint, transmission power lower bound, upper bound and the energy causality constraint respectively. Now we define \mathcal{D} as the set of $p_{i,k}$'s which satisfies (11) and (12). The Lagrange dual function is defined as: $g(\lambda) = \max_{\{p_{i,k}\} \in \mathcal{D}} \mathcal{L}(\{p_{i,k}\}, \lambda, \eta_{i,k}, \delta_{i,k}, \mu_{i,k})$, and the dual problem is defined as $\min_{\lambda \geq 0} g(\lambda)$. The Lagrange dual function $g(\lambda)$ can be obtained by solving the corresponding optimization problem using the following Karush-Kuhn-Tucker (KKT) conditions, where the optimal primal and dual solutions are denoted as $p_{i,k}^*, \mu_{i,k}^*, \delta_{i,k}^*, \eta_{i,k}^*$:

$$\begin{aligned} & \frac{T - \tau_k}{MT \log 2} \frac{h_{i,k} a_{i,k} (1 - \theta_k)}{1 + \sum_{l=1}^N p_{l,k}^* h_{l,k} a_{l,k} (1 - \theta_k)} + \eta_{i,k}^* - \delta_{i,k}^* \\ & - \frac{T - \tau_k}{MT} \lambda g_{i,k} a_{i,k} (1 - \theta_k) \\ & - \sum_{r=k}^M a_{i,r} (T - \tau_r) (1 - \theta_r) \mu_{i,r}^* = 0 \quad \forall i, k \\ & \eta_{i,k}^* p_{i,k}^* = 0 \quad \forall i, k \\ & \delta_{i,k}^* (p_{i,k}^* - P_{max}) = 0 \quad \forall i, k \\ & p_{i,k}^* \leq P_{max} \quad \forall i, k \end{aligned} \quad (14)$$

with $p_{i,k}^* \geq 0, \delta_{i,k}^* \geq 0, \eta_{i,k}^* \geq 0, \mu_{i,k}^* \geq 0$. Also $p_{i,k}^*,$

$\mu_{i,k}^*, \delta_{i,k}^*, \eta_{i,k}^*$ denote the optimal transmission power, optimal Lagrange parameters for energy harvesting constraint, transmission power upper bound and lower bound constraint respectively.

From the above mentioned KKT conditions we get the expression for $p_{i,k}^*$ by following the same approach proposed in [3]. The analytic derivation for the optimal sensing power is mentioned below.

Lemma 1: Let i and j be any two arbitrary users, $i, j \in 1, 2, \dots, N$, with $p_{i,k}^* > 0$ and $p_{j,k}^* = 0$, then the following must hold:

$$\frac{c_{i,k}}{\lambda d_{i,k} + e_{i,k}} \geq \frac{c_{j,k}}{\lambda d_{j,k} + e_{j,k}}, \quad \forall k \quad (15)$$

where $c_{i,k}, d_{i,k}, e_{i,k}$ are given by the following expressions: $c_{i,k} = \frac{T - \tau_k}{MT \log 2} h_{i,k} a_{i,k} (1 - \theta_k)$, $d_{i,k} = \frac{T - \tau_k}{MT} g_{i,k} a_{i,k} (1 - \theta_k)$, and $e_{i,k} = \sum_{r=k}^M \mu_{i,r}^* a_{i,r} (T - \tau_r) (1 - \theta_r)$.

Proof: The proof is given in Appendix A. ■

Now let π be a permutation over $\{1, 2, \dots, N\}$ such that $\frac{c_{\pi(i),k}}{\lambda d_{\pi(i),k} + e_{\pi(i),k}} \geq \frac{c_{\pi(j),k}}{\lambda d_{\pi(j),k} + e_{\pi(j),k}}$ when $i < j$, $i, j \in \{1, 2, \dots, N\}$. Suppose there are $|\mathcal{I}|$ users that can transmit, where $\mathcal{I} \subseteq \{1, 2, \dots, N\}$ denotes this set of users. It can be verified that $\mathcal{I} = \{\pi(1), \dots, \pi(|\mathcal{I}|)\}$. The following Lemma provides a way to determine the cardinality of the set \mathcal{I} .

Lemma 2: The cardinality of the set \mathcal{I} , $|\mathcal{I}|$ is the largest value of x such that

$$\begin{aligned} & \frac{c_{\pi(x),k}}{\lambda d_{\pi(x),k} + e_{\pi(x),k}} > \\ & 1 + \sum_{b=1}^{x-1} P_{max} h_{\pi(b),k} a_{\pi(b),k} (1 - \theta_k) \end{aligned} \quad (16)$$

and the optimal transmission power for $\pi(|\mathcal{I}|)^{th}$ user is given by the following expression:

$$\begin{aligned} p_{\pi(|\mathcal{I}|),k}^* = \min(P_{max}, & \left(\frac{c_{\pi(|\mathcal{I}|),k}}{\lambda d_{\pi(|\mathcal{I}|),k} + e_{\pi(|\mathcal{I}|),k}} \right. \\ & \left. - 1 - \sum_{b=1}^{|\mathcal{I}|-1} P_{max} h_{\pi(b),k} a_{\pi(b),k} \right) \frac{1}{h_{\pi(|\mathcal{I}|),k}} \end{aligned} \quad (17)$$

Proof: The proof is given in Appendix B. ■

Note that the solution for $p_{\pi(c),k}^*$ is P_{max} if $c < |\mathcal{I}|$. If $c > |\mathcal{I}|$, then $p_{\pi(c),k}^* = 0$. Only for the case when $c = |\mathcal{I}|$, then $p_{\pi(c),k}^*$ is given by (17).

As an example if we consider a specific case of $M = 2$ and number of SUs $N = 3$, then for any horizon if τ_k is fixed, the optimal transmission power would lie in the open set $(0, P_{max})$ for at most one of the 3 SUs. All the other SUs either transmit with P_{max} or do not transmit.

B. Optimal Sensing Time Policy

Fixing the transmission power would result in the optimization problem in $\{\tau_k\}$:

$$\begin{aligned} \max_{\{\tau_k\}} \quad & \sum_{k=1}^M \frac{T - \tau_k}{MT} \log_2\left(1 + \sum_{i=1}^N p_{i,k} h_{i,k} a_{i,k} (1 - \theta_k)\right) \\ \text{s.t.} \quad & \frac{1}{M} \sum_{k=1}^M \frac{T - \tau_k}{T} \sum_{i=1}^N p_{i,k} g_{i,k} a_{i,k} (1 - \theta_k) \leq Q_{avg} \\ & 0 \leq \tau_k \leq T; 1 \leq k \leq M \\ & \sum_{r=1}^k a_{i,r} (p_s \tau_r + p_{i,r} (T - \tau_r) (1 - \theta_r)) \leq B_i + \sum_{r=1}^{k-1} H_{i,r} \end{aligned}$$

The optimization problem can be simplified as follows:

$$\begin{aligned} & \min_{\boldsymbol{\tau}} \mathbf{s}^T \boldsymbol{\tau} \\ & \text{s.t. } \mathbf{w}^T \boldsymbol{\tau} \geq \tilde{Q}, \mathbf{0} \preceq \boldsymbol{\tau} \preceq T \cdot \mathbf{1}, \mathbf{Y}^T \boldsymbol{\tau} \succeq \mathbf{z} \end{aligned} \quad (18)$$

where $\mathbf{s}, \boldsymbol{\tau}, \mathbf{w}, \tilde{Q}, \mathbf{Y}, \mathbf{z}$ are given by the following expressions

$$\begin{aligned} s_k &= \log_2 \left(1 + \sum_{i=1}^N p_{i,k} h_{i,k} a_{i,k} (1 - \theta_k) \right) \\ \mathbf{s} &= [s_1, s_2, \dots, s_M]^T, \boldsymbol{\tau} = [\tau_1, \tau_2, \dots, \tau_M]^T \\ w_k &= \sum_{i=1}^N p_{i,k} g_{i,k} a_{i,k} (1 - \theta_k), \mathbf{w} = [w_1, w_2, \dots, w_M]^T \\ \tilde{Q} &= T \left(\sum_{k=1}^M w_k \right) - M Q_{avg}, \rho_{i,r} = a_{i,r} \{ p_{i,r} (1 - \theta_r) - p_s \} \\ \boldsymbol{\Gamma}_i &= \begin{bmatrix} \rho_{i,1} & 0 & 0 & 0 \\ \rho_{i,1} \rho_{i,2} & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & 0 \\ \rho_{i,1} \rho_{i,2} \cdots \rho_{i,M} \end{bmatrix}, \mathbf{Y} = [\boldsymbol{\Gamma}_1, \boldsymbol{\Gamma}_2, \dots, \boldsymbol{\Gamma}_N]^T \\ z_{i,k} &= \sum_{r=1}^k a_{i,r} p_{i,r} T (1 - \theta_r) - \left(B_i + \sum_{r=1}^{k-1} H_{i,r} \right) \\ \mathbf{z} &= [z_{1,1}, \dots, z_{1,M}, \dots, z_{N,1}, \dots, z_{N,M}]^T \end{aligned}$$

and $\mathbf{0}, \mathbf{1}$ are M dimensional vector of all 0's and 1's respectively. Problem (18) is a linear program, which can be solved using a standard LP solver with a reasonable accuracy and complexity.

Convergence analysis of alternating convex optimization: The objective function of the optimization problem in (5) is monotonically increasing during the alternating convex optimization procedure as at each iteration it is maximized with respect to a parameter by fixing the other parameter. Hence the value of the objective function either increases or remains unchanged. But it is important to note that even by considering only the short term constraints (7) and (8), and removing the average interference constraint in an extreme scenario, i.e., $a_{i,k} = 1, \forall i, k, \theta_k = 0, \forall k, \tau_k^* = 0, \forall k$ and $p_{i,k}^* = P_{max}, \forall i, k$, the sum capacity would be upper bounded by the expression $\mathbb{E} \left\{ \sum_{k=1}^M \frac{1}{M} \log_2 \left(1 + \sum_{i=1}^N P_{max} h_{i,k} \right) \right\}$, where the expectation is taken over $h_{i,k}$. Using Jensen's inequality, we can write:

$$\begin{aligned} & \mathbb{E} \left\{ \sum_{k=1}^M \frac{1}{M} \log_2 \left(1 + \sum_{i=1}^N P_{max} h_{i,k} \right) \right\} \\ & \leq \sum_{k=1}^M \frac{1}{M} \log_2 \left(1 + \sum_{i=1}^N P_{max} \mathbb{E} \{ h_{i,k} \} \right) \\ & = \sum_{k=1}^M \frac{1}{M} \log_2 \left(1 + \sum_{i=1}^N P_{max} \right) \end{aligned}$$

Thus the objective function is upper bounded by the sum capacity achievable by a feasible solution for $a_{i,k}, \theta_k, p_{i,k}, \tau_k$ for a relaxed constraint set, which indicates that a upper bounded monotonically non-decreasing function must converge. Since the original problem is non-convex, this convergence is only guaranteed to reach a local optimum.

IV. CAUSAL OPTIMIZATION WITH FINITE BATTERY

While the non-causal information pattern is unrealistic, it serves as a benchmark for the more realistic scenario of causal

CSI and battery state information with finite battery capacity. The resulting problem can be formulated as a finite horizon stochastic control problem and thus solved by a dynamic programming algorithm. The values of $a_{i,k}$ and θ_k , we can be found either via an exhaustive search or suboptimally by the previously discussed heuristic policy depending on the complexity concerned with a given scenario.

A. Information Pattern

During each time slot k , FC receives the CSI between SU transmitter and PU receiver $\mathbf{g}_k = \{g_{1,k}, g_{2,k}, \dots, g_{N,k}\}$ causally, either via cooperation from the PU receiver (or base station), or via feedback from a cooperative node located close to the PU receiver. The CSI between SU transmitter and FC $\mathbf{h}_k = \{h_{1,k}, h_{2,k}, \dots, h_{N,k}\}$ and the SU battery state information $\mathbf{B}_k = \{B_{1,k}, B_{2,k}, \dots, B_{N,k}\}$ are assumed to be received by FC via typical channel estimation techniques and feedback from the SUs to the FC. Information available to FC at k^{th} horizon is given by the tuple $\mathbf{J}_k = \{\mathbf{g}_k, \mathbf{h}_k, \mathbf{B}_k, \mathbf{J}_{k-1}\}$.

B. Dynamic Programming Algorithm

Now we discuss the process of finding the optimal *sensing time* and *transmission power* for energy harvesting SUs with finite battery to maximize the sum-capacity at FC under the assumption that the information about channel gain and energy arrival process of all SUs are causally available at the FC. We assume the sum-capacity expression corresponding to a fixed horizon k can be written as

$$\begin{aligned} C(p_{i,k}, \tau_k) &= \frac{T - \tau_k}{MT} \log_2 \left(1 + \sum_{i=1}^N p_{i,k} h_{i,k} a_{i,k} (1 - \theta_k) \right) \\ & - \lambda \left(\frac{T - \tau_k}{MT} \sum_{i=1}^N p_{i,k} g_{i,k} a_{i,k} (1 - \theta_k) - Q_{avg} \right) \end{aligned} \quad (19)$$

Here λ is the Lagrange parameter corresponding to average interference constraint.

We first define the feasible set for the optimization variables as:

$$S = \{(p_{i,k}, \tau_k) : p_{i,k}, \tau_k \text{ satisfy (7), (8), (9)}\}$$

With λ fixed, the optimal value of *transmission power* and *sensing time* can be determined by the following theorem:

Theorem 1. *With the initial condition $\mathbf{J}_1 = \{\mathbf{g}_1, \mathbf{h}_1, \mathbf{B}_1\}$, the value of the finite horizon finite battery problem with causal information is given by $V_1(\mathbf{g}_1, \mathbf{h}_1, \mathbf{B}_1)$, which can be computed by the backward Bellman dynamic programming equation:*

$$\begin{aligned} V_k(\mathbf{g}, \mathbf{h}, \mathbf{B}) &= \max_{(p_{i,k}, \tau_k) \in S} [C(p_{i,k}, \tau_k) + \\ & \mathbb{E}[V_{k+1}(\mathbf{g}_{k+1}, \mathbf{h}_{k+1}, \mathbf{B}_{k+1} | p_{i,k}, \tau_k)]] \end{aligned}$$

Proof: The proof can be obtained by the optimality conditions for the finite horizon stochastic control problem [13]. ■

The solution of the causal optimization problem, which can be computed numerically by searching over discretized values of the optimization variables, is obtained as:

$$\begin{aligned} \{p_{i,k}^*, \tau_k^*\} &= \operatorname{argmax}_{p_{i,k}, \tau_k \in S} [C(p_{i,k}, \tau_k) + \\ & \mathbb{E}[V_{k+1}(\mathbf{g}_{k+1}, \mathbf{h}_{k+1}, \mathbf{B}_{k+1} | p_{i,k}, \tau_k)]] \end{aligned}$$

The optimal value of λ from (19) is found by the solving the

following equation:

$$\lambda \left\{ \mathbb{E} \left\{ \sum_{k=1}^M \frac{T - \tau_k}{T} \sum_{i=1}^N p_{i,k} g_{i,k} a_{i,k} (1 - \theta_k) \right\} - Q_{avg} \right\} = 0 \quad (20)$$

We solve (20) using a bisection algorithm, in which

- The variables $p_{i,k}^\lambda$ and τ_k^λ are obtained based on a fixed λ from (20). Hence, we express the interference term corresponding to fixed λ as $G(\lambda) = \mathbb{E} \left\{ \sum_{k=1}^M \frac{T - \tau_k^\lambda}{T} \sum_{i=1}^N p_{i,k}^\lambda g_{i,k} a_{i,k} (1 - \theta_k) \right\} - Q_{avg}$.
- The parameter $\lambda^{(k)}$ denotes the k^{th} iteration of λ in the Bisection iterative procedure.
- Two initial points $\lambda_1^{(1)}$ and $\lambda_2^{(1)}$ are chosen from the feasible set $\Lambda = \{\lambda : \lambda > 0\}$, such that $\lambda_1^{(1)} G(\lambda_1^{(1)}) < 0$ and $\lambda_2^{(1)} G(\lambda_2^{(1)}) > 0$.
- The bisection method is continued until $|\lambda^{(k)} G(\lambda^{(k)})| < \epsilon$, where $\epsilon > 0$ is a predefined threshold of convergence.
- The Bisection method converges since the expression $G(\lambda)$ can be shown to be monotonically decreasing with λ .

Note that this procedure is performed offline purely based on the statistics of the channel gains and harvested energy information. Based on this procedure, the FC creates a lookup table for optimal values of $p_{i,k}$ and τ_k corresponding to discrete quantized values of $\mathbf{g}_k, \mathbf{h}_k, \mathbf{H}_k$. In real time FC receives the channel gains and battery states and check for closest quantization point in its lookup table. The optimal sensing and transmission power are fetched from the look-up table and sent to the individual SUs (via errorfree feedback links with negligible delay), which are then used by the SUs for sensing and information transmission.

V. SIMULATION RESULTS

In this section we present numerical results for the causal and non-causal CSI and battery state scenarios for the optimization problem under consideration. The channels $g_{i,k}, h_{i,k}$ are modeled as exponentially distributed random variables with unity mean unless otherwise stated. We assume the energy harvesting process is also an exponentially distributed random process with unity mean. The PU activity probability is set to $\kappa = 0.8$. The sensing channel signal to noise ratio (SNR) is assumed to be -15dB . The PU signal variance is taken to be $\sigma_s^2 = 1$ mW. In all simulation studies, we assume $N = 2$ SUs. The length of a time slot is taken as $T = 2$ ms. The probability of false alarm P_{fa} is taken to be 0.03. The sampling frequency is assumed to be 1 MHz and the normalized threshold of detection is assumed to be $\frac{\gamma}{\sigma_n^2} = 1.006$. This corresponds to a minimum sensing time limit $\tau_l = 0.1$ ms. For a fair comparison, this constraint on the minimum sensing time has been applied to all of the non-causal, causal and heuristic policy based methods. Transmission power is assumed to be upper bounded by $P_{max} = 1$ mW. The initial battery level for each SU is assumed to be 0.4 mW. For dynamic programming $\mathbf{g}_k, \mathbf{h}_k, \mathbf{H}_k$ are quantized into 5 different discrete levels. For the first two plots we vary the length of horizon among $M = 2, 3, 4$. In Fig.2, we plot the average sensing time for non-causal CSI and battery state scenario denoted by τ_{avg} with respect to average harvested energy denoted by μ_H , keeping the mean of the channel distribution denoted by μ_g and μ_h constant. The averages are taken over 50 Monte-Carlo simulations. From Fig.2, it is evident that average sensing time τ_{avg} decreases monotonically by increasing the number

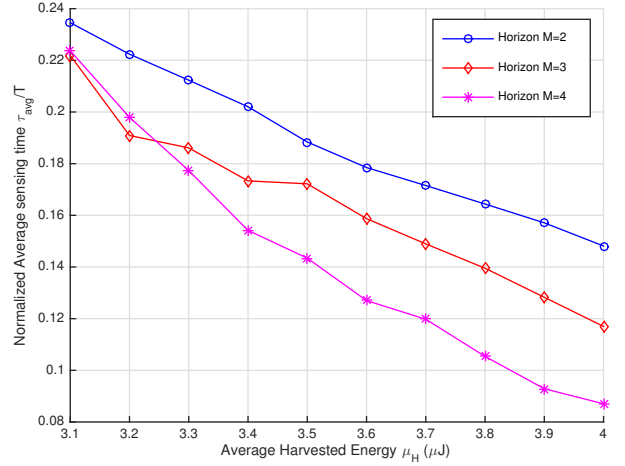


Figure 2: Normalized average sensing time τ_{avg} vs Average harvested energy μ_H with ratio of SU Tx-FC Rx and SU Tx-PU Rx average channel gain μ_h/μ_g fixed.

of horizons. This is due to the fact that increasing the length of horizon M in the non-causal CSI and battery state scenario helps to spread out the sensing time over multiple time slots, which means on average transmission time increases and sensing time decreases with increasing M . Fig.3 demonstrates of average sensing time τ_{avg} with respect to the ratio of channel mean μ_h/μ_g keeping μ_H fixed. The nature of the plot is similar to the Fig.2 as better SU direct channel gains compared to interference channel gains provide favourable transmission conditions which also increase transmission time and reduce sensing time. Fig.4 shows the optimized average throughput, i.e. average sum-capacity plotted against battery capacity B_{max} for non-causal CSI and battery state scenario with exhaustive search technique and causal CSI and battery state scenario with exhaustive search and heuristic policy. As expected the average throughput increases with increasing horizon and non-causal CSI and battery state scenario provides an upper bound for the causal counterpart. Also the heuristic policy by choosing $a_{i,k}$ and θ_k performs inferior to both of them, but serves as a less computationally complex alternative to its optimal counterpart. As a numerical comparison, it should be noted that for $M = 3$ and $B_{max} = 1\mu\text{J}$ the average throughput loss with exhaustive search and heuristic policy in causal CSI and battery state scenario compared to non-causal counterpart are 5.7 and 14.7 percent respectively. Fig.5 shows the heuristic policy-based optimized average throughput as a function of varying battery capacity in the causal CSI and battery state scenario with finite battery. These plots are obtained for three different horizons $M = 10, 15, 20$, significantly longer horizon lengths due to the reduced complexity of the heuristic policy.

VI. CONCLUSION

This paper investigated a MINLP problem of maximizing the expected achievable sum throughput in a fading multiple access CR network with the energy harvesting constraints where the CR nodes perform cooperative spectrum sensing. An analytical solution is obtained for the system with non-causal CSI and infinite battery capacity. This provides an upper bound on the throughput of a more realistic scenario involving causal CSI and finite battery capacity, which can

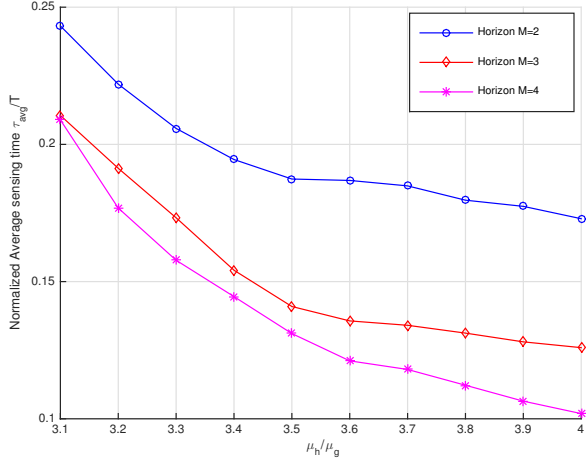


Figure 3: τ_{avg} vs ratio of SU Tx-FC Rx and SU Tx-PU Rx average channel gain μ_h/μ_g with average harvested energy μ_H fixed.

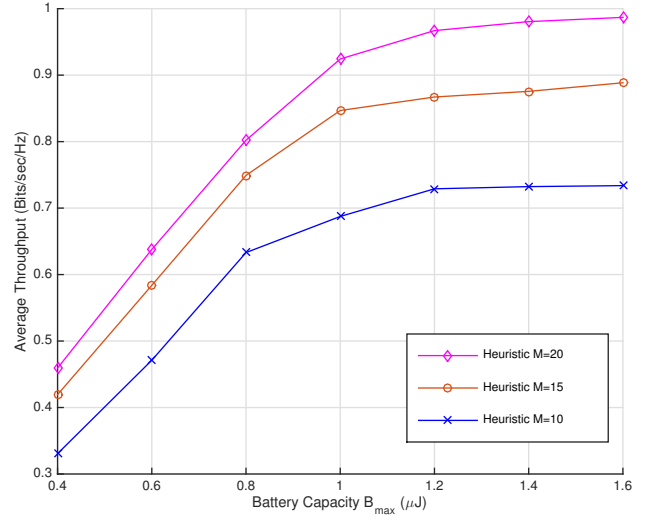


Figure 5: Average throughput vs B_{max} for the heuristic policy with $M = 10, 15, 20$.

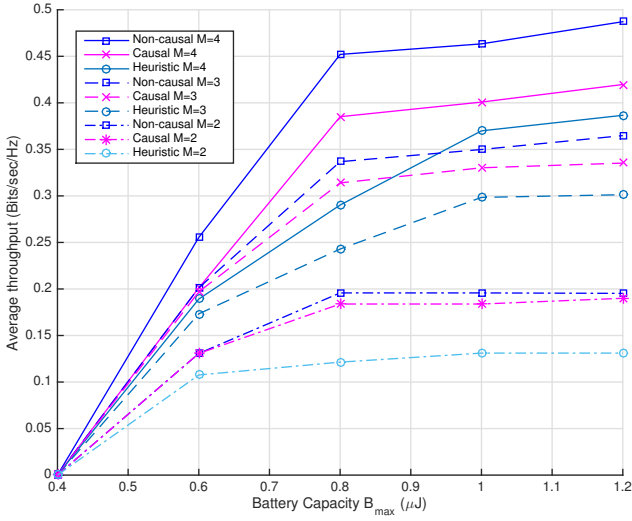


Figure 4: Average throughput vs B_{max} with causal and non-causal information pattern with $M = 2, 3, 4$.

be formulated as a stochastic control problem and solved using a dynamic programming algorithm. To combat the exponential complexity of handling exhaustive search policies involving the decision variables in the causal CSI scenario, a heuristic policy is proposed. The problem can be extended to incorporate the concept of infinite horizon optimization and energy sharing between the SUs, and also the innovation of better performing heuristic policies.

APPENDIX A PROOF OF LEMMA 1

Proof: Since $p_{j,k}^* = 0$ and $p_{i,k}^* > 0$ from the KKT condition it follows that $\delta_{j,k}^* = 0$ and $\eta_{i,k}^* = 0$. So it can

be deduced that:

$$\frac{\frac{T-\tau_k}{MT} h_{i,k} a_{i,k} (1-\theta_k)}{1 + \sum_{l=1}^N p_{l,k}^* h_{l,k} a_{l,k} (1-\theta_k)} - \frac{T-\tau_k}{MT} \lambda g_{i,k} a_{i,k} (1-\theta_k) - \sum_{r=k}^M \mu_{i,r}^* (T-\tau_r) a_{i,r} (1-\theta_r) \geq 0 \quad (21)$$

$$\frac{\frac{T-\tau_k}{MT} h_{j,k} a_{j,k} (1-\theta_k)}{1 + \sum_{l=1}^N p_{l,k}^* h_{l,k} a_{l,k} (1-\theta_k)} - \frac{T-\tau_k}{MT} \lambda g_{j,k} a_{j,k} (1-\theta_k) - \sum_{r=k}^M \mu_{j,r}^* (T-\tau_r) a_{j,r} (1-\theta_r) \leq 0 \quad (22)$$

From the above two equation the given lemma can be easily obtained. \blacksquare

APPENDIX B PROOF OF LEMMA 2

Proof: To prove it we have need the following lemma:

Lemma 3: The optimal solution of the problem has at most one user indexed by i that satisfies $0 < p_{i,k}^* < P_{max}$ where $i = \pi(|\mathcal{I}|)$, and the following condition must hold for the optimal transmission power:

$$\sum_{c=1}^{|\mathcal{I}|} p_{\pi(c),k}^* h_{\pi(c),k} a_{\pi(c),k} (1-\theta_k) = \frac{c_{\pi(|\mathcal{I}|),k}}{\lambda d_{\pi(|\mathcal{I}|),k} + e_{\pi(|\mathcal{I}|),k}} - 1 \quad (23)$$

Proof: (By contradiction) We assume that there exist two users i and j such that with $0 < p_{i,k}^* < P_{max}$ and $0 < p_{j,k}^* < P_{max}$. From the KKT condition we determine that $\delta_{i,k}^* = \delta_{j,k}^* = 0$ and $\eta_{i,k}^* = \eta_{j,k}^* = 0$ respectively. Using this values we can write:

$$\frac{c_{i,k}}{\lambda d_{i,k} + e_{i,k}} = \frac{c_{j,k}}{\lambda d_{j,k} + e_{j,k}} \quad (24)$$

Since $h_{i,k}$ and $g_{i,k}$'s are independent of $h_{j,k}$ and $g_{j,k}$'s and they are drawn from a continuous distribution and λ is

constant, it can be inferred that the above equality is satisfied with a probability zero. Thus we can deduce that there is at most one user i with $0 < p_{i,k}^* < P_{max}$. The optimal transmission power expression for the user i mentioned in the Lemma can be proven as the following:

Using the KKT condition and the expression of the proof it is easy to see that for any user $f \in \mathcal{I}, f \neq i$, with $p_{f,k}^* > 0$ must satisfy

$$\frac{c_{f,k}}{\lambda d_{f,k} + e_{f,k}} \geq \frac{c_{f,k}}{\lambda d_{f,k} + e_{f,k}} \quad (25)$$

Thus we can say that $i = \pi(|\mathcal{I}|)$.

From the previous Lemma, we can infer that there are only two possible sets of solutions for $p_{i,k}^*, k \in \mathcal{I}$:

- Case I: $p_{\pi(a),k}^* = P_{max}, a = 1, 2, \dots, |\mathcal{I}|$.
- Case II: $p_{\pi(a),k}^* = P_{max}, a = 1, 2, \dots, |\mathcal{I}| - 1$

The expression for $p_{\pi(|\mathcal{I}|),k}^*$ is only relevant when $a_{i,k} = 1$ and $\theta_k = 0$, which can be written as :

$$p_{\pi(|\mathcal{I}|),k}^* = \left(\frac{c_{\pi(|\mathcal{I}|),k}}{\lambda d_{\pi(|\mathcal{I}|),k} + e_{\pi(|\mathcal{I}|),k}} - 1 - \sum_{b=1}^{|\mathcal{I}|-1} P_{max} h_{\pi(b),k} a_{\pi(b),k} \right) \frac{1}{h_{\pi(|\mathcal{I}|),k}} \quad (26)$$

Since $p_{\pi(|\mathcal{I}|),k}^* \leq P_{max}$, thus we can write:

$$p_{\pi(|\mathcal{I}|),k}^* = \min(P_{max}, \left(\frac{c_{\pi(|\mathcal{I}|),k}}{\lambda d_{\pi(|\mathcal{I}|),k} + e_{\pi(|\mathcal{I}|),k}} - 1 - \sum_{b=1}^{|\mathcal{I}|-1} P_{max} h_{\pi(b),k} a_{\pi(b),k} \right) \frac{1}{h_{\pi(|\mathcal{I}|),k}} \quad (27)$$

The remaining part is showing the fact that optimal number of active users $|\mathcal{I}|$ is the largest value of x such that :

$$\frac{c_{\pi(x),k}}{\lambda d_{\pi(x),k} + e_{\pi(x),k}} > 1 + \sum_{b=1}^{x-1} P_{max} h_{\pi(b),k} a_{\pi(b),k} (1 - \theta_k) \quad (28)$$

It can be shown that both the case I and case II, any SU $\pi(b), b = 1, \dots, |\mathcal{I}|$, the above inequality holds. Since from Lemma 1, it can be said that its left hand side decreases with x while right hand side increases with x , thus it is sufficient to show that the inequality holds for $b = |\mathcal{I}|$. Thus in this case $\delta_{\pi(|\mathcal{I}|),k}^* \geq 0$ and $\eta_{\pi(|\mathcal{I}|),k}^* = 0$, we have

$$\begin{aligned} & \frac{c_{\pi(|\mathcal{I}|),k}}{\lambda d_{\pi(|\mathcal{I}|),k} + e_{\pi(|\mathcal{I}|),k}} \\ & \geq 1 + \sum_{b=1}^{|\mathcal{I}|} p_{\pi(b),k}^* h_{\pi(b),k} a_{\pi(b),k} (1 - \theta_k) \\ & > \sum_{b=1}^{|\mathcal{I}|-1} P_{max} h_{\pi(b),k} a_{\pi(b),k} (1 - \theta_k) \end{aligned} \quad (29)$$

Next we have to show that for any user $\pi(j), j = |\mathcal{I}| + 1, \dots, M$, the inequality does not hold. Again it is sufficient to show that it does not hold for $\pi(|\mathcal{I}| + 1)$. For that user

$\delta_{\pi(|\mathcal{I}|+1),k}^* = 0$ and $\eta_{\pi(|\mathcal{I}|+1),k}^* \geq 0$, we have

$$\begin{aligned} & \frac{c_{\pi(|\mathcal{I}|+1),k}}{\lambda d_{\pi(|\mathcal{I}|+1),k} + e_{\pi(|\mathcal{I}|+1),k}} \\ & \leq 1 + \sum_{b=1}^{|\mathcal{I}|} p_{\pi(b),k}^* h_{\pi(b),k} a_{\pi(b),k} (1 - \theta_k) \\ & \leq \sum_{b=1}^{|\mathcal{I}|} P_{max} h_{\pi(b),k} a_{\pi(b),k} (1 - \theta_k) \end{aligned} \quad (30)$$

Thus we can determine $|\mathcal{I}|$. ■

APPENDIX C PROOF OF LEMMA 4

Proof: This 2-horizon problem can be simplified as

$$\begin{aligned} \min_{\tau_1, \tau_2} & \quad s_1 \tau_1 + s_2 \tau_2 \\ \text{s.t.} & \quad w_1 \tau_1 + w_2 \tau_2 \geq \tilde{Q} \\ & \quad \tau_1 \leq \tau_1, \tau_2 \leq T \\ & \quad a_{i,1} (p_s \tau_1 + p_{i,1} (T - \tau_1) (1 - \theta(1))) \leq B_i \quad \forall i \\ & \quad a_{i,1} (p_s \tau_1 + p_{i,1} (T - \tau_1) (1 - \theta(1))) + \\ & \quad a_{i,2} (p_s \tau_2 + p_{i,2} (T - \tau_2) (1 - \theta(2))) \leq B_i + H_{i,1} \quad \forall i \end{aligned}$$

In the above equation the constants s_1, s_2, w_1, w_2 represents the following:

$$\begin{aligned} s_k &= \log_2 \left(1 + \sum_{i=1}^N p_{i,k} h_{i,k} a_{i,k} (1 - \theta_k) \right), \quad k = 1, 2 \\ w_k &= \sum_{i=1}^N p_{i,k} g_{i,k} a_{i,k} (1 - \theta_k), \quad k = 1, 2 \\ \tilde{Q} &= T(w_1 + w_2 - 2Q_{avg}) \end{aligned} \quad (31)$$

In this scenario, if we assume that the sensing time for the first horizon τ_1 is fixed, then the problem can be expressed as optimization over τ_2 . With this approach we have formulated the expression of τ_2 in terms of τ_1 .

Considering the energy causality constraint for the second horizon we get a lower bound of τ_2 in term of τ_1 as follows:

$$\begin{aligned} \tau_2 & \geq f_i(\tau_1) = \\ & \frac{T(a_{i,1} p_{i,1} (1 - \theta_1) + a_{i,2} p_{i,2} (1 - \theta_2)) - B_i - H_{i,1}}{a_{i,2} (p_{i,2} (1 - \theta_2) - p_s)} \\ & - \frac{a_{i,1} (p_{i,1} (1 - \theta_1) - p_s)}{a_{i,2} (p_{i,2} (1 - \theta_2) - p_s)} \quad \forall i \end{aligned}$$

Now as this inequality is valid for all users, we can write:

$$\tau_2 \geq f(\tau_1) = \max \{f_1(\tau_1), \dots, f_N(\tau_1)\} \quad (32)$$

Now another lower bound can be obtained form the interference constraint as the following:

$$\tau_2 \geq g(\tau_1) = \frac{\tilde{Q} - w_1 \tau_1}{w_2}$$

The highest lower bound of τ_2 is the optimal solution corresponding to the chosen fixed τ_1 , which can be written by the following expression:

$$\tau_2^* = \max \{f(\tau_1), g(\tau_1), \tau_1\} \quad \blacksquare$$

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