[3] R. Kumaresan and D. W. Tufts, "Estimating the angles of arrival of multiple plane waves," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-19, pp. 134-139, June 1983.
[4] M. A. Rahman and Y. K. Yu, "Total least square approach for frequency estimation using linear prediction," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-35, pp. 1440-1454, Oct. 1987.
[5] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," in *Proc. RADC Spectral Est. Workshop* (Griffiths AFBS, NY), 1979.
[6] G. H. Golub and C. V. Loan, *Matrix Computation*. Baltimore, MD: John Hopkins University Press, 1983.
[7] V. F. Pisarenko, "The retrieval of harmonics from a covariance function," *Geophys. J. Roy. Astron. Soc.*, vol. 33, pp. 347-366, 1973.
[8] S. M. Kay and A. K. Shaw, "Frequency estimation by principal component AR spectral estimation method without eigendecomposition," *IEEE Trans. Acoust., Speech, Signal, Processing*, vol. ASSP-36, pp. 95-101, Jan. 1988.
[9] R. Kumaresan, "Estimating the parameters of exponentially damped or undamped sinusoidal signals in noise," Ph.D. dissertation, Univ. Rhode Island, Kingston, RI, 1982.

Digital Differentiation of Noisy Data Measured Through a Dynamic System

Bengt Carlsson, Mikael Sternad, and Anders Ahlén

Abstract—**The design of discrete time differentiating filters, in the presence of colored noise and nonneglectable transducer dynamics, is investigated. The signal and noise are described by ARMA models, possibly with poles on $|z| = 1$. The MSE optimal filter, based on a discrete time approximation of the derivative operator, is given by a spectral factorization and a linear polynomial equation.**

I. Introduction

The problem of estimating derivatives of signals from noise corrupted measurements arises frequently. See, for example, [1], [2]. Several design principles have been suggested, see [3]-[7] for some recent approaches. In [3] and [4], this problem was addressed by means of stochastic signal models. Reference [3, sec. IV] presented an MSE optimal estimator design based on a discrete time model. The transducer dynamics affecting the measurements were assumed to be negligible. In some applications though, the transducers cannot be neglected without seriously degrading the estimation accuracy. For example, a heating rate estimate may be required, based on sampled temperature data from a sensor with bandwidth below the Nyquist frequency. In such cases, we would like to estimate the derivative of the input to the transducer, rather than the derivative of the measured signal itself. In this correspondence, the differentiation problem is formulated in discrete time, with the transducer dynamics taken into account. The optimal (MSE) solution is derived.

Signal and noise models will be allowed to be marginally stable, i.e., to have poles on $|z| = 1$. This enables us to treat drifting signals, described as autoregressive integrated moving average (ARIMA) models, which frequently occur in differentiation problems. Shape-deterministic signals, such as ramps which change derivative at random instants, can also be handled.

The purpose of this correspondence is thus to extend [3, sec. IV]. Moreover, a new methodology for deriving Wiener filter design equations, suggested in [9] for stationary data, is shown to be applicable (with small modifications) also for marginally stable signal models.

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II. Problem Formulation

Consider the following discrete time model, where transfer functions are expressed as ratios of polynomials in the backward shift operator ($q^{-1}y(k) = y(k-1)$):

$$y(k) = q^{-\tau}\frac{B(q^{-1})}{A(q^{-1})}u(k) + \frac{M(q^{-1})}{N(q^{-1})}v(k) \tag{2.1}$$

$$D(q^{-1})u(k) = C(q^{-1})e(k). \tag{2.2}$$

In this model, discussed in [8], $u(k)$ is the input to the transducer. The measurement $y(k)$ is affected by an additive colored noise process. The signals are scalar and real valued. Let $e(k)$ and $v(k)$ be white and mutually independent, with zero means and variances λ_e and λ_v, respectively. All polynomials, except $B(q^{-1})$, are monic.

Based on sampled measurements up to time $k + m$, we would like to estimate the nth order derivative of $u(k)$. Here, $m < 0$, $m = 0$, and $m > 0$ correspond to prediction, filtering, and fixed lag smoothing problems, respectively. The derivative is a continuous time concept. Following the lines of [3], a discrete time approximation of the derivative $d^n u(t)/dt^n$ is introduced as

$$d(k) = q^l \frac{S(q^{-1})}{T(q^{-1})} u(k). \tag{2.3}$$

This (fictitious) signal approximates the nth order derivative of an underlying continuous time signal $u(t)$, at the sampling instants. The approximation may be noncausal ($l > 0$). Note that l, S, and T in (2.3) are user choices. The frequency response of $q^l S/T$ should be close to $(i\omega)^n$ in the frequency band of interest. A simple choice for $n = 1$ is $q^l S/T = (q - q^{-1})/2h$, where h is the sampling interval. This choice give a good approximation for $\omega < 0.5/h$, say. See, for example, [3], [4], [10], [11] for a further discussion of different approximations.

Introduce a stable linear derivative estimator

$$\hat{d}(k|k+m) = \frac{Q(q^{-1})}{R(q^{-1})} y(k+m) \tag{2.4}$$

where $Q(q^{-1})$ and $R(q^{-1})$ are to be determined so that the MSE criterion

$$J = Ez(k)^2 = E(d(k) - \hat{d}(k|k+m))^2 \tag{2.5}$$

is minimized. See Fig. 1.

III. The Optimal Differentiating Filter

For $P(q^{-1}) = p_o + p_1 q^{-1} + \cdots + p_{np} q^{-np}$, of degree np, let $P_*(q)$ denote the conjugate polynomial $p_o + p_1 q + \cdots + p_{np} q^{np}$. Polynomials $P(z^{-1})$ are denoted stable if all their zeros are in $|z| < 1$. Factors with zeros only on $|z| = 1$ are denoted marginally stable. Arguments of polynomials will frequently be omitted, for brevity. Following [3], [8], and [9], we introduce the polynomial

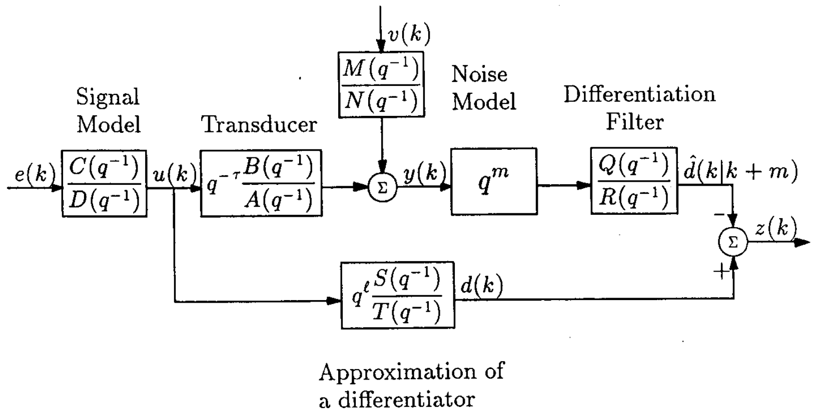

Fig. 1. The differentiation problem, with colored noise and a dynamic measurement system. The signal $d(k)$ is an approximation of the nth order derivative of $u(k)$. An MSE optimal linear estimator of $d(k)$ is to be designed.

spectral factorization

$$r\beta\beta_* = CBNC_*B_*N_* + \rho MADM_*A_*D_* \tag{3.1}$$

where r is a scalar, $\rho \triangleq \lambda_v/\lambda_e$, and $\beta(q^{-1})$ is a polynomial of degree

$$n\beta = \max(nc + nb + nn, nm + na + nd). \tag{3.2}$$

The following assumptions are made.

A1) The model denominators A, N, and D have zeros in $|z| \leq 1$.

A2) No common factors of CBN and MAD have zeros on or outside $|z| = 1$.

A3) The denominator T of the derivative approximation is chosen stable.

Under A2, (3.1) can be solved for a stable β. This implies detectability of an innovations model $y(k) = (\beta/DAN)\eta(k)$. (Marginally stable modes are then observable from $y(k)$. Corresponding denominator factors cannot be cancelled by factors of β, which are all stable.) The main result is presented in the following theorem.

Theorem 1: Assume A1–A3 to hold. The criterion (2.5) is globally minimized if and only if the filter transfer function in (2.4) has the same coprime factors as

$$\frac{Q}{R} = \frac{Q_1 NA}{\beta T} \tag{3.3}$$

where $\beta(q^{-1})$ is the stable spectral factor from (3.1) and where $Q_1(q^{-1})$, together with $L_*(q)$, is the unique solution to the linear "diophantine" polynomial equation

$$q^{l+\tau-m}SCC_*B_*N_* = Q_1 r\beta_* + qDTL_* \tag{3.4}$$

with degrees

$$nQ_1 = \max(nc + ns - l - \tau + m, nd + nt - 1)$$

$$nL = \max(nc + nb + nn + l + \tau - m, n\beta) - 1. \tag{3.5}$$

The minimal criterion value becomes

$$Ez(k)^2_{\min} = \frac{\lambda_e}{2\pi i}\oint_{|z|=1} \frac{LL_*}{r\beta\beta_*} + \rho\frac{SS_*}{TT_*}\frac{CMAC_*M_*A_*}{r\beta\beta_*}\frac{dz}{z}. \tag{3.6}$$

Proof: See Appendix. ■

Remarks:

• With (3.5), (3.4) corresponds to a linear system of equations with an equal number of unknowns and equations. It is nonsingular since $D(z^{-1})T(z^{-1})$ (with zeros in $|z| \leq 1$) and $\beta_*(z)$ (with zeros in $|z| > 1$) cannot have common factors. Thus, a unique solution to (3.4) exists. In problems where unstable D are allowed and where D and β_* have common factors, two coupled diophantine equations would be needed. For simplicity, we do not treat this case here since it is of very limited practical interest.

• The transfer function B/A is allowed to have poles on the unit circle. Transducers will, however, seldom have this property. A consequence of A2 is that N is not allowed to have common factors with either A or D with zeros on the unit circle.

• In certain special cases, some of our eariler results could be utilized. When the transducer dynamics can be neglected, $\tau = 0$ and $A = B = 1$. The problem and solution then reduce to the ones presented[1] in [3]. An alternative approach is to include the derivative approximation into C/D and its inverse into B/A. The problem is then reduced to a deconvolution problem, by making the substitutions $u(k) = d(k)$, $C = CS$, $D = DT$, $B = BT$, $A = AS$ and $\tau = \tau - l$. See [8, sec. IIIB] or [8, Example 2]. Unfortunately, assumption A2 would then seldom hold. Marginally stable or unstable common factors of the new C and A would be a frequent problem. They could, in general,[2] be avoided only for stable S. This would, however, exclude most realistic derivative approximations since S, in general, in not stable. No such problems occur in the solution (3.1)–(3.4).

• From the proof of Theorem 1, we note that the derivation technique suggested in [9] is applicable also for models with poles on $|z| = 1$. Exactly the same tricks can be used as in the nonconstructive proofs in [3] and [8]: stationarity of a variational term $n(k)$ is assured by a restriction on the zeros of admissible estimator variations. The stationarity of $z(k)$ rests on cancellation of marginally stable modes and is verified separately.

• There exist efficient numerical algorithms for performing polynomial (FIR) spectral factorization. Closed-form expressions exist for second-order spectral factors [12]. If the right-hand side

[1] There, S and T are denoted B and A.

[2] One exception would be when marginally stable factors of S are also factors of D or B in the original problem. They could then be cancelled in at least one block in the transformed problem, and a stable β would result.

of (3.1) is $g_o + g_1(q + q^{-1}) + g_2(q^2 + q^{-2})$, r and $\beta(q^{-1}) = 1 + \beta_1 q^{-1} + \beta_2 q^{-2}$ will be given by

$$\gamma \triangleq \frac{g_o}{2} - g_2 + \sqrt{\left(\frac{g_o}{2} + g_2\right)^2 - g_1^2}$$

$$r = (\gamma + \sqrt{\gamma^2 - 4g_2^2})/2; \qquad \beta_1 = \frac{g_1}{r + g_2}; \qquad \beta_2 = \frac{g_2}{r}. \tag{3.7}$$

IV. A Numerical Example

We will here give a simple example, to illustrate the design procedure. Assume that $u(k) = u(k-1) + e(k)$, i.e., $D(q^{-1}) = 1 - q^{-1}$ and $C(q^{-1}) = 1$, and

$$y(k) = \frac{1}{1 - 0.7q^{-1}} u(k-1) + \frac{1}{1 + 0.4q^{-1}} v(k) \tag{4.1}$$

with $\rho = \lambda_v/\lambda_e = 0.1$. As an approximation of a first-order derivative of $u(k)$, we use

$$d(k) = q(0.5 - 0.5q^{-2})u(k), \qquad (h = 1) \tag{4.2}$$

which has fairly good accuracy in the transducer passband. A 3-lag smoothing estimator (2.4) will be designed. Using (4.1), the spectral factorization (3.1) is

$$r\beta\beta_* = (1 + 0.4z^{-1})(1 + 0.4z) + 0.1(1 - 0.7z^{-1}) \cdot (1 - z^{-1})(1 - 0.7z)(1 - z)$$

with stable solution, from (3.7), $\beta(q^{-1}) = 1 + 0.0670q^{-1} + 0.0441q^{-2}$ and $r = 1.5878$. We obtain $nQ_1 = \max\{3, 0\} = 3$ and $nL = \max\{0, 2\} - 1 = 1$ from (3.5). Equation (3.4) is, using $m = 3$,

$$\begin{aligned} q^{1+1-3}&(1 + 0.4q)(0.5 - 0.5q^{-2}) \\ &= (Q_o + Q_1 q^{-1} + Q_2 q^{-2} + Q_3 q^{-3})1.5878(1 + 0.067q \\ &\quad + 0.0441q^2) + q(1 - q^{-1})(L_o + L_1 q). \end{aligned}$$

By equating terms of equal power of q, the solution $L_*(q) = -0.0383 - 0.0059q$, $Q_1(q^{-1}) = (1 - q^{-1})(0.0840 + 0.4198q^{-1} + 0.3149q^{-2})$ is obtained. Thus, the optimal differentiating smoothing filter becomes

$$\hat{d}(k|k+3) = \frac{Q_1(q^{-1})(1 + 0.4q^{-1})(1 - 0.7q^{-1})}{1 + 0.067q^{-1} + 0.0441q^{-2}} y(k+3). \tag{4.3}$$

The transfer function has a zero at $z = 1$. (By evaluating (3.4) at $z = 1$, this is seen to be the case whenever both D and S have zeros at $z = 1$.) Furthermore, there are zeros at the pole locations of the noise and transducer models. The minimal error variance is $0.0516\lambda_e$. See Figs. 2 and 3 for an illustration of the design.

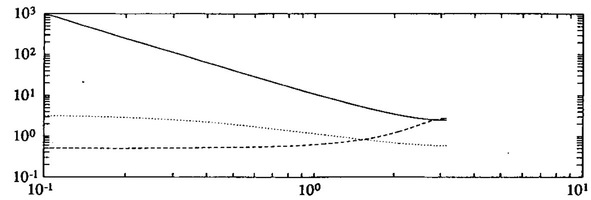

Fig. 2. Spectral densities of the signal $u(k)$ (solid line) and the measurement noise (dashed line). The transducer magnitude is also shown (dotted line).

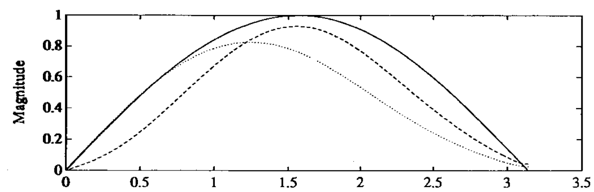

Fig. 3. Magnitudes of the derivative approximation S/T in (4.2) (solid line), optimal filter Q/R in (4.3) (dashed line), and transfer function of transducer and optimal filter BQ/AR (dotted line). Note that where the signal dominates over the noise, the magnitude of BQ/AR is close to the magnitude of S/T.

Appendix
Proof of Theorem 1

The constructive derivation methodology suggested in [9] is utilized. It is extended to cope with marginally stable systems. With (2.1)–(2.4), the estimation error is

$$z(k) = \left(q^l \frac{S}{T} - q^{m-\tau} \frac{Q}{R}\frac{B}{A}\right) \frac{C}{D} e(k) - q^m \frac{QM}{RN} v(k). \tag{A.1}$$

All admissible alternatives to a proposed estimate $\hat{d}(k|k+m)$, given by (2.4), can be described by $(Q/R)y(k+m) + n(k)$, where

$$n(k) = \mathcal{G}y(k+m) = \mathcal{G}\frac{\beta}{DAN}\eta(k+m). \tag{A.2}$$

Here $\mathcal{G}$ is a causal, stable, but otherwise arbitrary rational function, and η is the innovation. Optimality of (2.4) is asured if the corresponding error $z(k)$ is orthogonal to any admissible estimator variation $n(k)$, i.e., $Ez(k)n(k) = 0$.

Assume $E(z(k))^2$ to be finite. (This will be verified when the optimal estimator has been obtained.) Evidently, to keep a perturbed error variance (for $n(k) \neq 0$) finite, $E(n(k))^2$ must also be finite. Thus, $\mathcal{G}$ in (A.2) must be constrained so that its numerator *cancels all zeros of DAN on the unit circle*. Both $z(k)$ and $n(k)$ will then be stationary and Parseval's formula gives $Ez(k)n(k)$ as

$$\begin{aligned} &E\left\{\left(q^l \frac{S}{T} - q^{m-\tau}\frac{QB}{RA}\right)\frac{C}{D}e(k) - q^m \frac{QM}{RN}v(k)\right\} \\ &\quad \cdot\left\{q^{m-\tau}\mathcal{G}\frac{BC}{AD}e(k) + q^m \mathcal{G}\frac{M}{N}v(k)\right\} \\ &= \frac{\lambda_e}{2\pi i}\oint\left\{\left(z^{l-m+\tau}\frac{S}{T} - \frac{QB}{RA}\right)\frac{C}{D}\frac{B_*C_*}{A_*D_*} - \rho\frac{QMM_*}{RNN_*}\right\}\mathcal{G}_*\frac{dz}{z} \\ &= \frac{\lambda_e}{2\pi i}\oint\left\{\frac{z^{l+\tau-m}SCC_*B_*N_*NAR - TQr\beta\beta_*}{DTNARN_*A_*D_*}\right\}\mathcal{G}_*\frac{dz}{z}. \end{aligned} \tag{A.3}$$

In the last step, (3.1) was used. Now, $Ez(k)n(k) = 0$ is fulfilled if all poles in $|z| \leq 1$ are cancelled by zeros in (A.3). With our constraint on $\mathcal{G}$, $(1/N_*A_*D_*)\mathcal{G}_*$ will have poles only in $|z| > 1$. All other poles are in $|z| \leq 1$. Thus, we require

$$z^{l+\tau-m}SCC_*B_*N_*NAR - TQr\beta\beta_* = zL_*DTNAR$$

for some polynomial $L_*(z)$ or, equivalently,

$$(z^{l+\tau-m}SCC_*B_*N_* - zL_*DT)NAR = Qr\beta_*\beta T. \tag{A.4}$$

The right-hand side of (A.4) must contain R as a factor. We keep our options open by not assuming *a priori* that R and Q have common factors. Furthermore, since R must be stable, it cannot include factors of β_*. Thus, set $R = \beta T$, and cancel βT in (A.4). Observe that NA must be factor of Q, i.e., $Q = Q_1 NA$. Cancel NA and exchange q for z to obtain (3.4). With $R = \beta T$ and $Q = Q_1 NA$ (which may contain stable common factors), we have (3.3).

A unique solution $\{Q_1, L_*\}$ to (3.4) is guaranteed by (3.5) and the coprimeness of β_* and DT. The minimal variance (3.6) is obtained by inserting (3.1), (3.3), and (3.4), in this order, into (2.5). The "only if" part of the result follows because choices of Q/R other than (3.3) correspond to $n(k) \neq 0$. The criterion value would be $E(z(k) - n(k))^2 = E(z(k))^2 + E(n(k))^2$ since $Ez(k)n(k) = 0$. It would thus increase.

It remains to verify the stationarity and finite variance of $z(k)$. Insert (3.3) into (A.1) to obtain

$$z(k) = \left(q^l \frac{S}{T} - q^{m-\tau} \frac{Q_1 NAB}{\beta TA}\right) \frac{C}{D} e(k) - q^m \frac{Q_1 NAM}{\beta TN} v(k). \tag{A.5}$$

Cancellation of A and N is assumed to be exact in (A.5) and βT will be stable. It remains to show that poles on $|z| = 1$ in C/D must be cancelled by zeros. Denote the zeros of D by $\{z_o\}$. Note that when (3.1) and (3.4) are evaluated in $\{z_o\}$, their most right-hand terms vanish. Using (3.3), (3.4), and (3.1), the transfer function from $u(k)$ to $\hat{d}(k|k + m)$, evaluated in $\{z_o\}$, becomes

$$z_o^{m-\tau} \frac{Q_1 NB}{\beta T} = z_o^{m-\tau} z_o^{l+\tau-m} \frac{SCC_* B_* N_* NB}{r\beta\beta_* T} = z_o^l \frac{S}{T}.$$

It equals the transfer function from $u(k)$ to $d(k)$. Thus, the transfer function from $u(k)$ to $z(k)$ has zeros at all poles of C/D. Hence, $z(k)$ will be stationary. ∎

References

[1] H. J. Woltring, "On optimal smoothing and derivative estimation from noisy displacement data in biomechanics," *Human Movement Sci.*, vol. 4, pp. 229-245, 1985.

[2] F. Farina and F. A. Studer, *Radar Data Processing*, vol. 1. New York: Research Student Press and Wiley, 1985.

[3] B. Carlsson, A. Ahlén, and M. Sternad, "Optimal differentiation based on stochastic signal models," *IEEE Trans. Signal Processing*, vol. SP-39, pp. 341-353, Feb. 1991.

[4] B. Carlsson, "Digital differentiating filters and model based fault detection," Ph.D. dissertation, Uppsala Dissertations from the Faculty of Science 28, Uppsala University, Uppsala, Sweden, 1989.

[5] S.-C. Pei and J.-J. Shyu, "Design of FIR Hilbert transformers and differentiators by eigenfilters," *IEEE Trans. Circuit Syst.*, vol. CS-35, pp. 1457-1461, Nov. 1988.

[6] M. R. R. Reddy, B. Kumar, and S. C. Dutta Roy, "Design of efficient second and higher order FIR digital differentiators for low frequencies," *Signal Processing*, vol. 20, pp. 219-225, July 1990.

[7] B. Carlsson, "Maximum flat digital differentiator," *Electron. Lett.*, vol. 27, pp. 675-677, Apr. 1991.

[8] A. Ahlén and M. Sternad, "Optimal deconvolution based on polynomial methods," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 37, pp. 217-226, Feb. 1989.

[9] A. Ahlén and M. Sternad, "Wiener filter design using polynomial equations," *IEEE Trans. Signal Processing*, vol. 39, no. 11, pp. 2387-2399, Nov. 1991.

[10] L. R. Rabiner and K. Steiglitz, "The design of wide-band recursive and nonrecursive differentiators," *IEEE Trans. Audio Electron.*, vol. AU-18, pp. 204-209, June 1970.

[11] J. Spriet and J. Bens, "Optimal design and comparisons of wide-band digital on-line differentiators," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-27, pp. 46-52, Feb. 1979.

[12] V. Peterka, "Predictor-based self-tuning control" *Automatica*, vol. 20, pp. 39-59, Jan. 1984.

A Fast Finite-State Algorithm for Vector Quantizer Design

Ruey-Feng Chang, Wen-Tsuen Chen, and Jia-Shung Wang

***Abstract*—The Linde-Buzo-Gray (LBG) algorithm is usually used to design a codebook for encoding images in the vector quantization. In each iteration of this algorithm, we must search the full codebook in order to assign the training vectors to their corresponding codewords. Therefore, the LBG algorithm needs a large computation effort to obtain a good codebook from the training set. In this correspondence, we propose a finite-state LBG (FSLBG) algorithm for reducing the computation time. Instead of searching the entire codebook, we search only those codewords that are close to the codeword for a training vector in the previous iteration. In general, the number of these possible codewords can be very small without sacrificing performance. Because of searching only a small part of the codebook, the computation time is reduced. In our experiment, the performance of the FSLBG algorithm in terms of the signal-to-noise ratio is very close to that of the LBG algorithm. However, the computation time of the FSLBG algorithm is only about 10% of the time required by the LBG algorithm.**

I. Introduction

In recent years, vector quantization (VQ) has been found to be an efficient technique for image compression [1], [2]. One major advantage of VQ is that the hardware structure of the encoder, and especially the decoder, is very simple. The images to be encoded are first processed to yield a set of vectors. Then a codebook is generated using, for example, the iterative clustering algorithm proposed by Linde, Buzo, and Gray [3]. The input vectors are then individually quantized to the closest codewords in the codebook. Compression is achieved by using the indices of codewords for transmission or storage. Reconstruction of the images can be implemented by the table lookup techniques; the indices are simply used as addresses to the corresponding codewords in the codebook.

The key step in vector quantization is to generate a good codebook from the training image. The K-means [4] and the closely related generalized Lloyd clustering algorithm proposed by Linde, Buzo, and Gray [3], (LBG algorithm), are typically used to generate the codebook. These algorithms are basically iterative processes to minimize the distortions between the training vectors and their corresponding codewords. A major disadvantage of the LBG algorithm is that a large effort is required for searching the entire codebook in each iteration to find the closest codeword for a training vector. In this correspondence, we propose a finite-state LBG (FSLBG) algorithm to reduce the execution time of designing a codebook. This algorithm searches only a part of the codebook, not the whole codebook, to find the corresponding codeword for a

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