INT. J. CONTROL, 1991, VOL. 53, NO. 3, 579–596

Use of disturbance measurement feedforward in LQG self-tuners

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An explicit adaptive regulator with disturbance measurement feedforward is presented, based on a polynomial LQG design. The addition of an optimized feedforward filter to a feedback regulator involves the solution of only one additional linear polynomial equation. The regulator is designed to handle shape-deterministic disturbances, such as steps, ramps and sinusoids, as well as stochastic disturbances. The properties of the off-line solution in the case of unstable disturbance models are explained. Computational aspects, the computational complexity and the robustness against unmodelled dynamics are discussed. It is argued that the use of feedforward can improve not only the disturbance rejection, but also the stability robustness of an LQG feedback regulator.

1. Introduction

A feedforward regulator utilizes measurements of important disturbances. The regulator can react to the disturbance *before* it begins to affect the controlled variable. Complete disturbance cancellation may sometimes be achieved. Addition of feedforward filters to feedback regulators is a simple way to improve the control performance, at moderate extra computational cost.

LQG optimization is a useful framework for the design of combined feedback and feedforward regulators. It provides trade-offs between input energy and disturbance rejection. Control of discrete-time systems with input delays or non-minimum phase dynamics becomes straightforward. Several alternative approaches do however exist, such as generalized minimum variance control (GMV) (see Åström and Wittenmark 1973, Clarke and Gawthrop 1979, Allidina *et al.* 1981, Tahmassebi *et al.* 1985). Compared to adaptive algorithms based on infinite horizon LQG criteria, GMV often attains inferior asymptotic performance. This is the case in particular for non-minimum phase systems (compare Modén and Söderström 1982 and Sternad 1987). The quest for improved performance has led to the modification of GMV into generalized predictive control (GPC) (Clarke *et al.* 1987, Perez and Kershenbaum 1986). As the prediction horizon increases, the performance of GPC approaches that of infinite horizon LQG control, from below.

LQG optimization can be based on polynomial equations (Kučera 1979). The polynomial equations approach to the design of combined feedback–feedforward regulators has received considerable interest recently (see Peterka 1984, Šebek *et al.* 1988, Sternad and Söderström 1988, Grimble 1988 b, Hunt 1989). LQG self-tuners with disturbance-measurement feedforward have been proposed by Sternad (1986, 1987), Hunt *et al.* (1987) and Hunt and Šebek (1989 a).

The purpose of this paper is to discuss the following aspects related to off-line and adaptive feedback–feedforward control, based on LQG design.

Received 23 October 1989. Revised 27 April 1990.

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(*a*) While LQG design is based on a stochastic disturbance description, random-step sequences, ramp sequences and sinusoids can also be handled. The properties of the solution in the case of unstable disturbance models will be discussed in some detail. (Such problems have also been discussed by Hunt 1989, who utilized an alternative proof technique.) Disturbance-measurement feedforward can be combined with integrating feedback, although some care must be taken with the regulator calculation and implementation.

(*b*) Feedforward control may be used to improve the stability robustness of feedback regulators. Assume that a given amount of disturbance rejection is desired. The regulator design is based on an uncertain and/or underparametrized model. When most of a disturbance can be eliminated by feedforward, the high-frequency gain of the feedback can be reduced. The feedback can be designed to maintain robust stability, rather than high disturbance rejection. (With an incorrect model, the feedforward control performance will of course be non-ideal, but this can never destabilize the system.)

The paper is organized as follows. For the control problem, defined in § 2, the polynomial LQG solution is presented in § 3. A self-tuning implementation is described in § 4. Some user-choices, which affect the robustness of the control law, are discussed in § 5. A previous version of this paper was presented at the IFAC Symposium on Adaptive Systems in Control and Signal Processing (ACASP-89) in Glasgow in April 1989.

2. Control problem

Let the plant be described by the following linear discrete-time model

$$A(q^{-1})y(t) = B(q^{-1})u(t-k) + D(q^{-1})w(t-d) + C(q^{-1})n(t) \tag{2.1}$$

where the output $y(t)$, input $u(t)$, measurable disturbance $w(t)$ and unmeasurable disturbance $n(t)$ are all scalar signals. All model polynomials, of degree n_a, n_b and so on, are expressed in the backward shift operator q^{-1}. All, except $B(q^{-1})$ and $D(q^{-1})$, are monic. The delays are $k > 0$ and $d \geqslant 0$.

The disturbances $w(t)$ and $n(t)$ are modelled by

$$\left.\begin{aligned} w(t) &= \frac{G(q^{-1})}{H(q^{-1})}v(t) = \frac{G(q^{-1})}{H_S(q^{-1})H_U(q^{-1})}v(t) \\ n(t) &= \frac{1}{F(q^{-1})}e(t) \end{aligned}\right\} \tag{2.2}$$

We assume $v(t)$ and $e(t)$ to be mutually uncorrelated and zero mean. They are stationary white noises or random spike sequences, with variance λ_v and λ_e, respectively. While $C(q^{-1})$, $G(q^{-1})$ and $H_S(q^{-1})$ are assumed to be stable, $H_U(q^{-1})$ and $F(q^{-1})$ have all their zeros on the unit circle. (Disturbance models with poles strictly outside the unit circle are not considered. They are of limited interest, since regulation of exponentially increasing disturbances would be doomed to failure in practice.) The disturbance models thus include:

(*a*) Stationary stochastic disturbances (F or $H_U = 1$);

(*b*) drifting stochastic disturbances. If $w(t)$ has, for example, stationary increments, it is modelled by $H_U = 1 - q^{-1}$ and a white noise $v(t)$;

(*c*) shape-deterministic or piecewise deterministic signals, such as random step sequences, ramp sequences or sinusoids which occasionally change magnitude or phase. A stationary random spike sequence, such as a Bernoulli–gaussian sequence, is then a reasonable model for $v(t)$ or $e(t)$. (A Bernoulli–gaussian sequence is given by $v(t) = r(t)s(t)$ where $s(t)$ is Bernoulli sequence such that $s(t) = 1$ with probability λ and $s(t) = 0$ with probability $1 - \lambda$. $r(t)$ is a zero mean gaussian sequence with variance σ^2 independent of t, see Mendel 1983. It is then straightforward to show that $v(t)$ is a stationary white sequence with zero mean and variance $\lambda_v = \sigma^2\lambda$.)

Assume, for now, that all polynomials are known. The goal is to minimize the infinite horizon criterion

$$J = \lim_{N \to \infty} \frac{1}{N} \sum_{t=0}^{N} Ey(t)^2 + \rho E(W(q^{-1})F(q^{-1})u(t))^2 \tag{2.3}$$

The input penalty $\rho \geqslant 0$ and the polynomial $W(q^{-1})$ are chosen by the designer. They define a frequency-dependent trade-off between input energy and disturbance rejection. Note that the choice of input filter is not completely free: the factor $F(q^{-1})$ must be present whenever $n(t)$ is described by an unstable model. If $n(t)$, for example, is a drifting stochastic signal, a drifting input $u(t)$ will be needed. To keep the criterion finite, the input must then be filtered by $F(q^{-1}) = 1 - q^{-1}$ in (2.3).

We may consider yet another type of disturbance, as follows.

(*d*) Deterministic signals. Such measurable disturbances are described by autonomus difference equations $H(q^{-1})w(t) = 0$, with non-zero initial values. Optimal control of the disturbance types (*a*), (*b*) and (*c*) above results in outputs with finite (and in general non-zero) power. Control of a deterministic disturbance would, however, result in only one initial transient, with finite energy and zero power measured for $t \in [0, \infty)$. The criterion (2.3) would then be zero. To include deterministic signals in our framework, they are treated as shape-deterministic. Their transient phase, which is to be optimized, is formally considered to be repeated. We use the model $H(q^{-1})w(t) = v(t)$, where $v(t)$ is a random spike sequence.

It can be shown (Sternad 1987) that the optimal linear regulator structure, with feedback and feedforward, is given by

$$R(q^{-1})F(q^{-1})u(t) = -\frac{Q(q^{-1})}{P(q^{-1})} w(t) - S(q^{-1})y(t) + m(t) \tag{2.4}$$

(see Fig. 1). The external signal $m(t)$ is set to zero in the following. The polynomial $P(q^{-1})$ is required to be stable. Note that the filter $1/R(q^{-1})F(q^{-1})$ is present in both the feedback and feedforward signal paths. The filtering by $1/R(q^{-1})F(q^{-1})$ is consistent with the internal model principle (Francis and Wonham 1976). When $F(q^{-1}) = 1 - q^{-1}$, we have an integrating regulator with a feedforward term.

Complete elimination of the measurable disturbance can be achieved if and only if $d \geqslant k$ and all unstable factors of $B(q^{-1})$ are also factors of $D(q^{-1})$. (The parts of the system which cause non-minimum phase behaviour are then located beyond the point where the disturbance meets the control action).

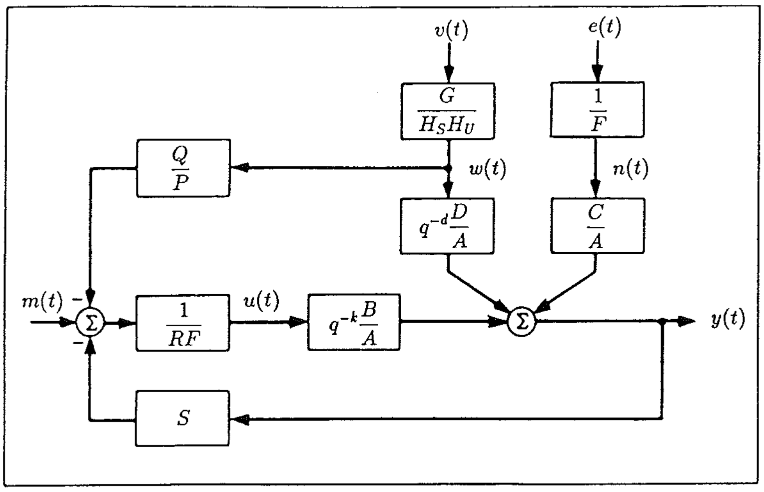

Figure 1. System and regulator structure.

Sternad and Söderström (1988) presented a polynomial equation by which the feedforward filter $\{P, Q\}$ can be optimized, given *any* stabilizing feedback $\{R, S\}$. The use of PID-control, optimal feedback or no feedback at all are some examples. Stable disturbance models ($H_U = 1, F = 1$) were assumed.

In this paper, we discuss the optimization of the total regulator (2.4), allowing $H_U \neq 1$ and/or $F \neq 1$. The design consists of a simple two-step procedure: the feedback $\{R, S\}$ is first optimized with respect to the unmeasurable disturbance $n(t)$. The feedforward filter $\{P, Q\}$ is then calculated so that $w(t)$ is rejected in an optimal way. This separability is made possible by the use of the regulator structure (2.4) and by the (assumed) non-correlation between $w(t)$ and $n(t)$. The feedback is a well-known result (Kučera 1979), slightly generalized to cover $F \neq 1$.

3. Optimal regulator

Let us adopt the following polynomial notation. For any polynomial in the backward shift operator q^{-1}, of degree n_u,

$$U(q^{-1}) = u_0 + u_1 q^{-1} + \dots + u_{n_u} q^{-n_u}$$

Let $U_*(q) \triangleq u_0 + u_1 q + \dots + u_{n_u} q^{n_u}$ and $\bar{U}(q^{-1}) \triangleq q^{-n_u} U_*(q) = u_0 q^{-n_u} + u_1 q^{-n_u+1} + \dots + u_{n_u}$. In the frequency domain, the complex argument z is substituted for q. The polynomial arguments (q^{-1}, q, z^{-1}, z) will often be omitted. Stable polynomials $U(z^{-1})$ have all zeros in $|z| < 1$. If $U(z^{-1})$ is stable, $\bar{U}(z^{-1})$ will be unstable.

Introduce the polynomial spectral factorization

$$r\beta\beta_* = BB_* + \rho AFWW_*F_*A_* \tag{3.1}$$

where r is a positive scalar and $\beta(z^{-1})$ is a stable monic polynomial with degree n_β. When $\rho > 0$, the stability of β is assured if B and AFW have no common factors with zeros on the unit circle. If $\rho = 0$, B should have no zeros on the unit circle.

The following assumptions are sufficient for the existence of a unique stabilizing solution to the optimization problem described above.

(*a*) Polynomials β, C and G are stable;
(*b*) Polynomials AF and B have no unstable common factors;
(*c*) H_{U} is a factor of WFD.

Theorem

Under conditions (*a*)–(*c*) above, the controlled system (2.1), (2.4) attains the global minimum of (2.3), under the constraint of stability, if $\{R, S, P, Q\}$ are calculated as follows.

Let $R(z^{-1})$, $S(z^{-1})$ and $X_*(z)$ be the unique solution of the coupled linear polynomial equations

$$r\beta_* R - z^{-k+1}BX_* = \rho WW_* F_* A_* C \tag{3.2}$$

$$r\beta_* S + zAFX_* = z^k B_* C \tag{3.3}$$

Let $P = G$ and let $Q(z^{-1})$ and $L_*(z)$ be the unique solution of

$$z^{-d+1}DFGX_* = r\beta_* Q + zCHL_* \tag{3.4}$$

Proof

For the proof, see the Appendix.

Remark 1: *Optimization of feedback*

The variables in (3.2) and (3.3) have degrees ($n_x \triangleq \deg X$ and so on):

$$\left.\begin{aligned} n_x &= n_\beta + k - 1 \\ n_s &= \max\{n_f + n_a - 1, n_c - k\} \\ n_r &= \begin{cases} \max\{n_b + k - 1, n_c + n_w\} & \text{if } \rho \neq 0 \\ n_b + k - 1 & \text{if } \rho = 0 \end{cases} \end{aligned}\right\} \tag{3.5}$$

Linear polynomial equations have an infinite number of solutions, in general. That (3.2) and (3.3) have a unique solution, with polynomial degrees (3.5), is a consequence of R and S having to be polynomials only in z^{-1}, while X_* must be a polynomial in z. The degrees are defined by the requirement that the variables should cover the maximal occurring powers of z^{-1} and z in the equations.

Remark 2: *Interpretation of Conditions* (*a*)–(*c*)

Multiply (3.2) by AF and (3.3) by $z^{-k}B$ and add them. Optimal feedback is then seen to imply pole placement in βC:

$$AFR + z^{-k}BS = \beta C \tag{3.6}$$

In addition, the feedforward filter introduces poles in the zeros of $P = G$. This explains condition (*a*). If AF and B have no common factors, an optimal feedback can be calculated from the implied pole placement equation (3.6). With common factors, this will not be possible (Kučera 1984). The equations (3.2) and (3.3) do, however, give the correct solution, as long as the common factors of AF and B are stable.

The absence of unstable common factors of AF and B corresponds to stabilizability and detectability of an equivalent system, with $Fu(t)$ as input, obtained by multiplication of (2.1) by F:

$$(AF)y(t) = q^{-k}B(Fu(t)) + q^{-d}(DF)w(t) + Ce(t)$$

The condition that H_U divides WFD has a natural explanation. To be more specific, let $w(t)$ with $H_U(q^{-1}) = 1 - q^{-1} \triangleq \Delta(q^{-1})$ be a drifting stochastic disturbance, that is, $w(t) = w(t-1) + (G/H_S)v(t)$. The controller must then eliminate drifts from the signals $y(t)$ and $WFu(t)$, which appear in the criterion. Otherwise, the criterion would be infinite.

(*a*) If $H_U = \Delta$ is a factor of D, the non-stationarity of $w(t)$ is blocked and does not result in a drifting $y(t)$. Since Δ is a factor of D and H in (3.4), it will also be a factor of Q. Thus, the control signal $(Q/P)w(t)$ is stationary.

(*b*) If $H_U = \Delta$ is a factor of F, we have an integrating feedback regulator. It eliminates drifts in $y(t)$ caused by drifts in $w(t)$ (or in $n(t)$), regardless of the presence of any feedforward filter. The signal $Fu(t)$ (but not $u(t)$) is then stationary.

(*c*) If $H_U = \Delta$ is not a factor of DF, the responsibility for eliminating drifts in $y(t)$ is placed on the feedforward filter. To accomplish this task, it generates a drifting control signal $(Q/P)w(t)$. Consequently, Δ must be a factor of W, so that $Wu(t)$ is stationary and gives a finite contribution to the criterion.

When the disturbance $w(t)$ is described by a model with poles on the unit circle, such effects on $y(t)$ (e.g. static errors, drifts or undamped sinusoids) can be controlled either by the feedback or by the feedforward action. Use of the feedback, when H_U divides F, results in a robust disturbance rejection. The criterion is finite if modelling errors in A, B, C or D are present, as long as the closed-loop system is stable. When feedforward action is used, the magnitude of static errors, drifts or sinusoids can be reduced, but cannot be eliminated completely in practice. Modelling errors will cause imperfect cancellation.

Consider the case $F = \Delta$. In the disturbance description (2.2), $F = \Delta$ models the dynamics of steps and Wiener processes $n(t)$. In the regulator $F = \Delta$ represents integration. Nothing prevents us from using integration, i.e. to set $F = \Delta$ in (2.4)–(3.4), also when $n(t)$ is stationary. We cannot then attain the minimal criterion value, because the regulator has incorrect structure, but it may be advantageous to use integration anyway. When the feedforward filter is imperfectly designed, static control errors will then be taken care of by the feedback.

Remark 3: *Feedforward controller calculation*

Note that the solution of only one additional diophantine equation, namely (3.4), is needed for optimizing a feedforward filter. Since β (stable) and $z^{nc}C_* z^{nh}H_* = \overline{CH}$ (unstable) cannot have common factors, (3.4) is always solvable. The degrees of $Q(z^{-1})$ and $L_*(z)$ are defined uniquely by the requirement that they should cover the maximal occurring powers of z^{-1} and z, respectively, in (3.4):

$$\left.\begin{aligned} n_Q &= \max\{n_d + n_f + n_g + d, n_c + n_h\} - 1 \\ n_L &= \max\{0, k - d\} + n_\beta - 1 \end{aligned}\right\} \qquad (3.7)$$

The polynomial L_* is not used in the controller.

The delay d affects the achievable control quality significantly. It can be shown that application of feedforward can always improve the control performance when $d > 0$, compared to feedback from $y(t)$ only. The improvement is a non-decreasing function of d. It is advantageous to place the auxiliary $w(t)$-sensor so that the disturbance is captured as early as possible, i.e. d is large.

If the measurement $w(t)$ is influenced by the input $u(t-m), m > 0$, this effect could be subtracted internally, inside the regulator (see Sternad 1986, 1987). It is straightforward to generalize the solution to multiple measurable disturbances. One additional scalar diophantine equation (3.4) is then obtained for each disturbance. Feedforward control of multi-input systems using the matrix fraction descriptions, has been discussed by Hunt and Šebek (1989 a, b).

Remark 4: *Numerical aspects*

A common special case is when the measurable disturbance is drifting or of random step type, and an integrating regulator is used. Then, $H_U = F = \Delta$. Since Δ becomes a factor of both the left-hand side and rightmost term in (3.4), it must also be a factor of Q. With $Q = Q_1 \Delta$, (3.4) is reduced to

$$z^{-d+1} DGX_* = r\beta_* Q_1 + zCH_S L_* \tag{3.8}$$

In this case, the controller (2.4) must be modified slightly. It can be implemented in differential form, using an explicit differentiation of the measurable disturbance:

$$\left.\begin{aligned} R(\Delta u(t)) &= -\frac{Q_1}{G}(\Delta w(t)) - Sy(t) \\ u(t) &= u(t-1) + \Delta u(t) \end{aligned}\right\} \tag{3.9}$$

Alternatively, one can use a structure with the feedforward filter separated from the integration:

$$Ru(t) = -\frac{Q_1}{G} w(t) - \frac{S}{\Delta} y(t) \tag{3.10}$$

If (3.4) were used, small numerical errors and finite word-length effects would cause $Q \neq Q_1 \Delta$. This could lead to large errors in the low-frequency gain of the feedforward filter $-Q/R\Delta P$ in (2.4). Design from (3.8), with $n_{Q_1} = nQ - 1$, and realization according to (3.9) or (3.10), avoids such problems. Equation (3.4) must, however, be used in the general case, when $H_U \neq F$. The regulator should be realized minimally, as a single dynamical system having two inputs and one output.

A reliable algorithm for polynomial spectral factorization can be found in the work of Kučera (1979). It is iterative, requiring typically 3–10 iterations, when starting from $\beta = 1$. In adaptive control, β from the previous controller calculation can be used as the initial value. Then, normally only 1–2 iterations are required.

The coupled equations (3.2), (3.3) represent an over-determined set of simultaneous equations in the coefficients of R, S and X. The system will, however, have a unique solution. (Some equations are linear combinations of the others.) This (exact) solution can be found by computing the least-squares solution to the overdetermined system. Equation (3.4), with polynomial degrees (3.7), corresponds to a square system of linear equations, with full rank.

Example 1

Consider the system

$$(1-0{\cdot}9q^{-1})y(t)=(0{\cdot}1+0{\cdot}08q^{-1})u(t-2)+(0{\cdot}2+0{\cdot}4q^{-1})w(t-2)+e(t)$$

with $w(t)=w(t-1)+v(t)$, i.e. $H=1-q^{-1}$. Unit step disturbances $w(t)$ cause output deviations with amplitude 6 in this system. We design a regulator (2.4), such that the criterion (2.3), with differential input penalty $\rho=0{\cdot}1$ and $W=1-q^{-1}$, is minimized.

The spectral factorization (3.1) becomes

$$r(1+\beta_1 z^{-1}+\beta_2 z^{-2})(1+\beta_1 z+\beta_2 z^2)=(0{\cdot}1+0{\cdot}08z^{-1})(0{\cdot}1+0{\cdot}08z)$$
$$+0{\cdot}1(1-0{\cdot}9z^{-1})(1-z^{-1})(1-z)(1-0{\cdot}9z)$$

with solution $r=0{\cdot}2670$, $\beta_1=-0{\cdot}9887$ and $\beta_2=0{\cdot}3370$.

The feedback part of the regulator is calculated from (3.2) and (3.3):

$$r(1+\beta_1 z+\beta_2 z^2)R(z^{-1})-z^{-1}(0{\cdot}1+0{\cdot}08z^{-1})X_*(z)=0{\cdot}1(1-z^{-1})(1-z)(1-0{\cdot}9z)$$
$$r(1+\beta_1 z+\beta_2 z^2)S(z^{-1})+z(1-0{\cdot}9z^{-1})X_*(z)=z^2(0{\cdot}1+0{\cdot}08z)$$

The variables have degree $n_x=3$, $n_s=0$ and $n_r=2$, given by (3.5). Multiply the first equation by $z^{-n_\beta}=z^{-2}$ and the second by $z^{-n_x-1}=z^{-n_\beta-k}=z^{-4}$. We then obtain equations in powers of z^{-1} only.

$$r(\beta_2+\beta_1 z^{-1}+z^{-2})R(z^{-1})-(0{\cdot}1+0{\cdot}08z^{-1})\bar{X}(z^{-1})=0{\cdot}1(1-z^{-1})(z^{-1}-1)(z^{-1}-0{\cdot}9)$$
$$r(\beta_2+\beta_1 z^{-1}+z^{-2})z^{-2}S(z^{-1})+(1-0{\cdot}9z^{-1})\bar{X}(z^{-1})=z^{-1}(0{\cdot}1z^{-1}+0{\cdot}08)$$

By considering terms in equal powers of z^{-1}, a system of simultaneous equations, with block-Toepliz structure and with 10 equations and 8 unknowns, is obtained.

$$\left(\begin{array}{ccc|c|cccc}
r\beta_2 & 0 & 0 & 0 & -0{\cdot}1 & 0 & 0 & 0\\
r\beta_1 & r\beta_2 & 0 & 0 & -0{\cdot}08 & -0{\cdot}1 & 0 & 0\\
r & r\beta_1 & r\beta_2 & 0 & 0 & -0{\cdot}08 & -0{\cdot}1 & 0\\
0 & r & r\beta_1 & 0 & 0 & 0 & -0{\cdot}08 & -0{\cdot}1\\
0 & 0 & r & 0 & 0 & 0 & 0 & -0{\cdot}08\\
\hline
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & -0{\cdot}9 & 1 & 0 & 0\\
0 & 0 & 0 & r\beta_2 & 0 & -0{\cdot}9 & 1 & 0\\
0 & 0 & 0 & r\beta_1 & 0 & 0 & -0{\cdot}9 & 1\\
0 & 0 & 0 & r & 0 & 0 & 0 & -0{\cdot}9
\end{array}\right)
\begin{pmatrix} r_0\\ r_1\\ r_2\\ s_0\\ x_3\\ x_2\\ x_1\\ x_0 \end{pmatrix}
=
\left(\begin{array}{c}
0{\cdot}09\\ -0{\cdot}28\\ 0{\cdot}29\\ -0{\cdot}10\\ 0\\ \hline 0\\ 0{\cdot}08\\ 0{\cdot}1\\ 0\\ 0
\end{array}\right)$$

The solution is

$$R(z^{-1})=1-0{\cdot}08871z^{-1}+0{\cdot}1210z^{-2}$$
$$S(z^{-1})=s_0=1{\cdot}3617$$
$$X_*(z)=0{\cdot}4040+0{\cdot}04945z+0{\cdot}08z^2+0z^3$$

In specific examples, the polynomial degrees may be lower than the values indicated by (3.5) and (3.7). This is evident here, where X_* actually only has degree two.

Note that, in general, the $R(z^{-1})$ obtained will be non-monic, i.e. $r_0 \neq 1$. In this specific example, R is monic.

The feedforward polynomial $Q(z^{-1})$ is obtained from (3.4):

$$z^{-1}(0{\cdot}2 + 0{\cdot}4z^{-1})X_*(z) = r(1 + \beta_1 z + \beta_2 z^2)Q(z^{-1}) + z(1 - z^{-1})L_*(z)$$

with $X_*(z)$ from above, and with degress $n_Q = 2, n_L = 1$ from (3.7). The solution is

$$Q(z^{-1}) = 1{\cdot}8609 + 0{\cdot}9750z^{-1} + 0{\cdot}6052z^{-2}$$

$$L_*(z) = 0{\cdot}2521 - 0{\cdot}1675z$$

Thus, the optimal regulator (2.4) is

$$u(t) = 0{\cdot}08871u(t-1) - 0{\cdot}1210u(t-2) - 1{\cdot}3617y(t)$$
$$- 1{\cdot}8609w(t) - 0{\cdot}9750w(t-1) - 0{\cdot}6052w(t-2)$$

This regulator eliminates a unit-step disturbances $w(t)$, after a small initial transient with peak value 0·16, without excessive input variations.

4. LQG self-tuner

For systems with unknown or time-varying dynamics, an explicit LQG self-tuner has been developed by the author (Sternad 1987). It is based on recursive system identification using the recursive prediction error method (RPEM) (see Ljung and Söderström 1983). The controller is redesigned periodically, according to the Theorem. A similar algorithm has been suggested by Hunt *et al.* (1987) and (1989 a). Upper bounds on all polynomial degrees are assumed known, together with the unstable disturbance model factor $F(q^{-1})$. The regulator, complemented with a servo filter, is summarized below.

Step 1. Read new samples of $y(t)$, $w(t)$ and a set-point $r(t)$.

Step 2. Update models of $y(t)$ and $w(t)$ with the structure

$$\hat{A}y(t) = \hat{B}u(t) + \hat{D}w(t) + \hat{C}\varepsilon_y(t) \qquad (4.1)$$

$$\hat{H}w(t) = \hat{G}\varepsilon_w(t) \qquad (4.2)$$

using two RPEM routines for single output systems.

Step 3. Computer r and $\beta(q^{-1})$ from the spectral factorization (3.1).

Step 4. Determine $R(q^{-1})$, $S(q^{-1})$ and $X_*(q)$ from (3.2), (3.3).

Step 5. Calculate $Q(q^{-1})$ (and $L_*(q)$) from (3.4).

Step 6. If needed, design a servo filter $T(q^{-1})/E(q^{-1})$.

Step 7. Compute the control action:

$$RFu(t) = -\frac{Q}{\hat{G}}w(t) - Sy(t) + \frac{T}{E}r(t) \qquad (4.3)$$

Step 8. Shift all data vectors, and go to Step 1.

Remark 5

In Step 2, the regressors of the model (4.1) are filtered by

$$\frac{F(q^{-1})}{N(q^{-1})} \tag{4.4}$$

where $N(q^{-1})$ is a stable polynomial. Filtering by $F(q^{-1})$ is necessary to avoid biased estimates. With $N(q^{-1})$, the filter can be modified to improve the estimation accuracy in important frequency regions. The estimates of G and C must be projected into stable regions. The usual precautions of a control error dead-zone and covariance monitoring have been implemented. They guard against estimator wind-up and identification based on insufficient information. Time-varying systems and disturbances are tracked using forgetting factors.

Remark 6

In Step 6, the servo filter T/E can be designed by cancelling poles and stable zeros, so that the controlled system approximates a response model $y_m(t) = (q^{-k}B_m/A_m)r(t)$. This works well, but results in a rather high-order filter. Other approaches, such as including the servo design in the optimization, have been discussed by the authors elsewhere (Sternad 1987).

Remark 7

In Step 7, when appropriate, the regulator (3.9) or (3.10), based on (3.8), should be used instead of (4.3).

Global convergence of explicit LQG self-tuners can be demonstrated, under idealized conditions (see, for example, Chen and Gou 1986 or Grimble 1988 a). In general, a linear model structure (4.1), (4.2) cannot be expected to describe the true system exactly. Mismodelling is inevitable, to some extent. A good estimate of $q^{-k}B/A$, in the frequency ranges where the input has significant energy, is needed to assure stability. Errors in the estimates of the transfer functions $q^{-d}D/A$, C/A or G/H will affect the control performance, but they cannot cause instability. (Since the stability of $\hat{C}$ is monitored, pole placement in $\beta\hat{C}$ results in a stable system, if A and B are estimated correctly.)

Step 2. Identification		$37n^2$	$+36n$
Step 3. Spectral factorization (per iteration)		$3n^2$	$+3n$
Step 4. Feedback optimization	$36n^3$	$+87n^2$	$+135n$
Step 5. Feedforward optimization	$9n^3$	$+14n^2$	
Step 7. Control			$8n$

Approximate number of multi–add operations required per sample, assuming all model polynomials to have equal degree n. A least-squares solution is computed in Step 4.

The computational burden of this algorithm is significantly higher than for GMV (see the Table). With modern microcomputers and signal processors, this should be no significant restriction in most control applications. There is no need to recalculate the regulator at each sample. Steps 3–6 can be placed in a background process, which provides a new regulator every mth sample. For $m = 5$–10, this results in only a small degradation of the adaptation transient when the system dynamics changes. (It has recently been shown by Shimkin and Feuer 1988 that it may be advantageous to update the regulator infrequently.)

The behaviour of the algorithm is illustrated by some examples.

Example 2

Let $[1/(1-q^{-1})]v(t)$ be a square-wave disturbance, with unit amplitude and period 60. It disturbs the system

$$(1-0{\cdot}5q^{-1})y(t) = (b_2+b_3q^{-1})u(t-2)+(1+2q^{-1})w(t-1)$$

$$w(t)=\frac{1-0{\cdot}3q^{-1}}{1-0{\cdot}9q^{-1}}\left(\frac{1}{1-q^{-1}}v(t)\right)$$

The polynomial $b_2+b_3q^{-1}$ changes from $1+0{\cdot}1q^{-1}$ to $0{\cdot}5+0{\cdot}05q^{-1}$ at time 300. The LQG self-tuner, with correctly parametrized models, is applied. An input penalty $\rho=0{\cdot}5$ and $W=1-q^{-1}$, with $F=1$ is used, that is the feedback uses no integration. The forgetting factor is 0·98 in both RPEM algorithms. After an initial open-loop identification period of 20 samples, the regulator quickly converges.

Example 3

Consider the unstable and non-minimum phase system

$$(1-2q^{-1}+1{\cdot}5q^{-2})y(t)=(1+2q^{-1}+2q^{-2})u(t-1)+(1+0{\cdot}5q^{-1})w(t-2)$$

where $w(t)$ is white noise with standard deviation 0·1. As reference for the

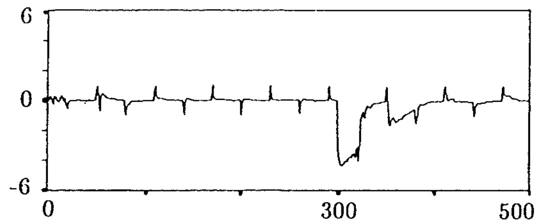

Figure 2. Controlled output $y(t)$ in Example 2. The disturbance $w(t)$ is cancelled almost completely, although the delay difference $k-d=1$ prevents perfect cancellation. At $t=300$, the system gain is halved. At $t=400$, the control performance has recovered to the off-line optimal one.

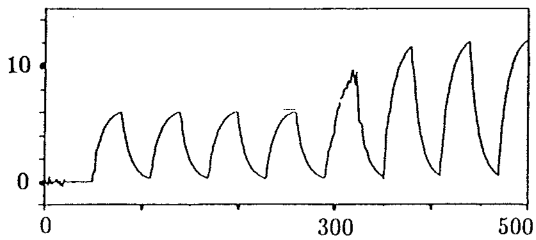

Figure 3. Input $u(t)$ in Example 2. When the system gain is halved at time 300, the regulator modifies itself, so that its gain is doubled.

controlled output,

$$y_m(t) = \frac{0{\cdot}7}{1 - 0{\cdot}3q^{-1}} r(t)$$

was used, with $r(t)$ being a square wave. Adaptation, with a correctly parametrized model, and with $\rho = 0$, started at $t = 1$. The regulator had essentially converged to off-line optimal control after 30 samples. Because the system is non-minimum phase, and the unstable part of B is not a factor of D, complete cancellation of the disturbance cannot be achieved (see Fig. 4). (Of course, it would not be advisable to start an adaptive regulator from scratch on such a system in practice; there is no guarantee that the signals behave acceptably in the transient phase.)

5. User choices affecting the robustness

The robustness against unmodelled dynamics of a self-tuner is affected by properties of both the estimator and the control law. Simple considerations regarding the LQG control strategy, which in general improve the robustness of both off-line and self-tuning designs, are illustrated by the following example.

Example 4

The system

$$(1 - 1{\cdot}2q^{-1} + 0{\cdot}52q^{-2})y(t) = q^{-2}(1 + 0{\cdot}8q^{-1})u(t) + q^{-2}(1 - 0{\cdot}2q^{-1})w(t) + (1 - 0{\cdot}2q^{-1})n(t)$$

is affected by measurable and unmeasurable drifting stochastic disturbances

$$w(t) = w(t-1) + v(t)$$

$$n(t) = n(t-1) + e(t)$$

The white noises $v(t)$ and $e(t)$ have standard deviations 0·3 and 0·1, respectively. Thus, the largest disturbance is measurable, and $H_U = F = \Delta$.

The control error standard deviation was measured (after convergence) in simulation runs with four self-tuners. Integrating regulators with the structure (3.9), with $r(t) = 0$ and $W = 1$, were used. The results are shown in Fig. 5, as functions of the input penalty ρ. Curve (1) represents the performance of LQG feedback and feedforward. When $\rho \to 0$, the disturbance $w(t)$ is cancelled completely by the feedforward control action. When only feedback is used, curve (2) is obtained. The disturbances $w(t)$ and $n(t)$ are then treated as one unmeasurable noise. The performance is obviously degraded without disturbance measurement. Correctly parametrized models were used in these experiments. The performance in each case was indistinguishable from the off-line optimum.

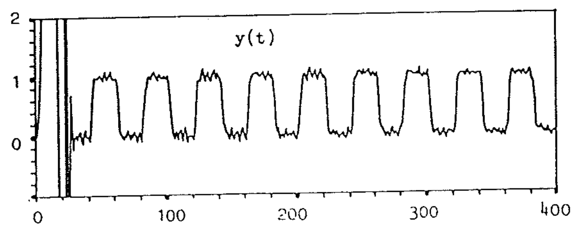

Figure 4. Unstable non-minimum phase system in Example 3, controlled by the LQG self-tuner.

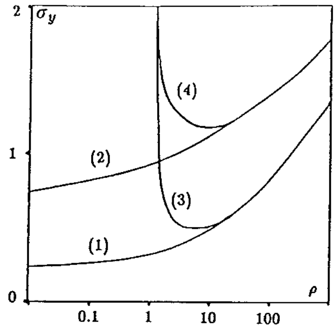

Figure 5. Output standard deviation σ_y in Example 4, as a function of the input penalty ρ: (1) feedback and feedforward, $\hat{B}$ of correct order three; (2) feedback only, $\hat{B}$ of correct order three; (3) feedback and feedforward, $\hat{B}$ of order two; (4) feedback only, $\hat{B}$ of order two.

Curves (3) and (4) result when an under-parametrized $\hat{B}$ is used. (Degree two instead of three, including the delay.) For input penalties $\rho \leqslant 1$, the closed-loop system then becomes unstable. The reason for this behaviour is explained by Figs 6 and 7. Figure 6 shows Bode magnitude plots of some under-parametrized models, obtained at the end of the simulation runs. Compare these with the true system. The high-frequency properties of the system are badly estimated. For low ρ, the regulators have large feedback gains at high frequencies (compare with Fig. 7). (This is often the case for minimum-variance regulators.) The combination of large feedback gain and an incorrect model at high frequencies leads to instability.

One way of reducing the high-frequency feedback gain is to modify the polynomial $C(q^{-1})$, used in (3.2)–(3.4). Instead of the estimate $\hat{C}$, a fixed polynomial $C_0 = (1 - 0{\cdot}5q^{-1})^2$ was used. This decreased the feedback high-frequency gain

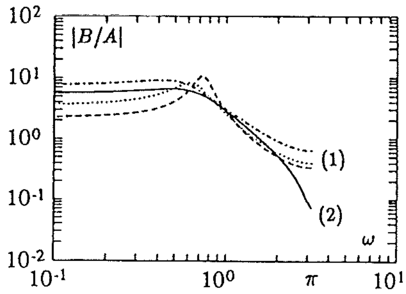

Figure 6. (1) Transfer function magnitudes for some under-parametrized models; (2) true system.

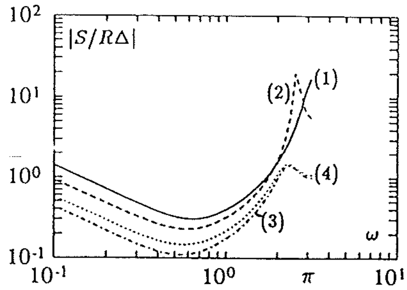

Figure 7. Transfer function magnitudes of feedback filters: (1) $\rho = 0$, (2) $\rho = 0{\cdot}5$, (3) $\rho = 10$, (4) $\rho = 0{\cdot}5$, with pole placement in C_0.

(see (4) in Fig. 7). The performance for high ρ deteriorated, but the robustness for low ρ improved (Fig. 8). The regulator now remained stable for $\rho \geqslant 0{\cdot}2$.

5.1. *Summary*

Let us summarize three robustness-enhancing user choices as follows.

(*a*) By increasing ρ from zero, the control signal variations and the high-frequency gains of both feedback and feedforward filters are reduced. Large reductions can often be achieved, with only a minor deterioration of the disturbance rejection. This increases the robustness against unmodelled high-frequency dynamics. Problems with hidden inter-sample output oscillations are also avoided. (Such oscillations are caused by pole placement on the negative real axis, which is often a consequence of minimum variance control.)

(*b*) Use of feedforward can increase the stability robustness. This is possible when high disturbance rejection is required, and the main system disturbance is measurable. For example, consider Fig. 5. If $\sigma_y < 1$ is required, this could be attained, in the ideal case (2), by a high gain (low ρ) feedback. Instability would, however, result in the under-parametrized case (4). With both feedback and feedforward, a low gain regulator ($\rho = 10-300$) can be used. It easily attains the required performance, also in the under-parametrized case.

(*c*) With LQG control, poles are placed in the zeros of βC (compare (3.6)). The polynomial C could be interpreted as the observer dynamics in a state space formulation. Use of a fixed prespecified observer polynomial C_0, with $1/C_0$ being low-pass, has several advantages. While the zeros of β can be modified via ρ, we do not have any control over the zero locations of C in the true system. (The admissible zero locations of discrete-time models of continuous-time stochastic processes have been investigated by Söderström 1989.) Furthermore, the coefficients of C are the hardest ones to estimate. Estimated C-polynomials sometimes tend to contain a factor $1 - q^{-1}$, which gives bad pole placement. (This happens when regressors are differentiated, but the disturbance $n(t)$ is not generated by a system with $F = 1 - q^{-1}$.) With a suboptimal pole placement βC_0, the feedback disturbance rejection may deteriorate. This matters less if feedforward can be applied. Compare the difference between curves (1) and (3) to that between curves (2) and (4) in Fig. 8.

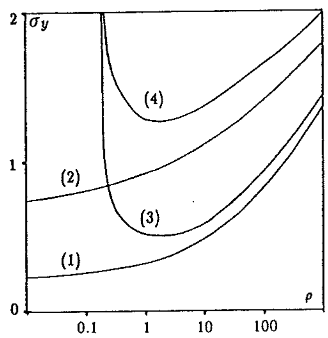

Figure 8. σ_y versus the input penalty ρ, when estimated polynomials $\hat{C}$ and the fixed prespecified $C_0 = (1 - 0{\cdot}5q^{-1})^2$ are used for pole placement. (1) Feedback and feedforward, $\hat{C}$ used, $\hat{B}$ of order three; (2) feedback only, $\hat{C}$ used, $\hat{B}$ of order three; (3) feedback and feedforward, C_0 used, $\hat{B}$ of order two; (4) feedback only, C_0 used, $\hat{B}$ of order two.

6. Conclusions

An explicit adaptive controller with disturbance measurement feedforward has been presented. It is based on polynomial LQG design and is capable of handling non-stationary and deterministic disturbances. The properties of the off-line solution when disturbance models are unstable have been studied. The roles played by the input penalty, the observer polynomial and feedforward control in determining a compromise between ideal-case performance and robustness have been exemplified.

In simulation studies, the adaptive algorithm has been found to behave very well in general. Compared to the explicit criterion minimization approach (Trulsson and Ljung 1985), the convergence rate of LQG self-tuners is much faster (see Sternad 1987). One (seldomly occurring) remaining problem is that over-parametrized models may contain unstable common factors of $\hat{A}$ and $\hat{B}$. The testing of schemes (such as that of De Laminat 1984) to avoid this is a problem for further research.

ACKNOWLEDGMENT

This paper is based on work supported by the Swedish National Board of Technical Developement under contracts 84-3680 and 88-02549.

Appendix

Proof of the Theorem

It will be verified that if (3.2), (3.3) and (3.4) are satisfied, the solution is admissible and optimal. First, the stability of the closed-loop system (excluding the disturbance models) is verified; then it is shown that the cost is finite. Finally, it is demonstrated that the attained cost is minimal. In that step, a technique previously used by Åström and Wittenmark (1984) when discussing feedback regulators with stationary disturbances is utilized.

Stability

Use of (2.1) and (2.4) gives the closed-loop system

$$y(t) = \frac{1}{\alpha}\left(\frac{M}{P} w(t) + FRCn(t) + q^{-k}Bm(t)\right) \tag{A 1}$$

$$u(t) = \frac{1}{\alpha}\left(-\frac{U}{P} w(t) - SCn(t) + Am(t)\right) \tag{A 2}$$

where

$$\alpha \triangleq AFR + q^{-k}BS$$

$$M \triangleq q^{-d}DFPR - q^{-k}BQ$$

$$U \triangleq q^{-d}DSP + AQ$$

Since $P = G$ and (3.6) gives $\alpha = \beta C$, the closed-loop system is stable.

Finite cost

Signals appearing in the criterion must have finite variance, even though $w(t)$ and $n(t)$ may be non-stationary. Let J_1 denote the cost (2.3) when the regulator

(2.4)–(3.4), with $m(t) = 0$, is applied. With (2.2), $P = G$, (A 1) and (A 2), it can be expressed as

$$J_1 = \lim_{N\to\infty} \frac{1}{N} \sum_{t=0}^{N} E(y_v(t) + y_e(t))^2 + \rho E(z_v(t) + z_e(t))^2 \tag{A 3}$$

where

$$z_v(t) + z_e(t) \triangleq WFu(t)$$

$$y_v(t) = \frac{M}{\alpha G}\frac{G}{H_S H_U} v(t), \qquad y_e(t) = \frac{FRC}{\alpha}\frac{1}{F} e(t)$$

$$z_v(t) = -\frac{WFU}{\alpha G}\frac{G}{H_S H_U} v(t), \quad z_e(t) = -\frac{WFSC}{\alpha}\frac{1}{F} e(t)$$

Both $y_e(t)$ and $z_e(t)$ are stationary and have finite variance, since the cancellation of the unstable factor F is assumed to be exact.

The signal $y_v(t)$ has finite variance if (and only if) the unstable denominator factor H_U divides M. Consider the polynomial $r\beta_* M$. By using first (3.4) and then (3.2), it can be expressed as

$$\begin{aligned} r\beta_* M &= q^{-d}DFGr\beta_* R - q^{-k}Br\beta_* Q \\ &= q^{-d}DFGr\beta_* R - q^{-d}DFGq^{-k+1}BX_* + q^{-k+1}BCHL_* \\ &= q^{-d}DFG\rho WW_* F_* A_* C + q^{-k+1}BCH_S H_U L_* \end{aligned}$$

Since H_U is assumed to be a factor of DFW, it will be a factor of $\beta_* M$. However, β_* has no zeros on the unit circle, so H_U, which has all its zeros on the unit circle, must be a factor of M. Consequently, $y_v(t)$ has finite variance.

Factors of H_U which are factors of WF are cancelled in the expression for $z_v(t)$. Common factors of H_U and D must, according to (3.4), also be factors of Q. (They cannot be factors of β_*, since β_* is assumed to have no zeros on the unit circle.) Thus, such factors are factors of $U = q^{-d}DSG + AQ$. Consequently, $z_v(t)$ is stationary, with finite variance, if H_U is a factor of WFD.

Optimality

Let an arbitrary (but not destabilizing) control action be expressed as (2.4), where R, S, P and Q are calculated according to the Theorem, and $m(t)$ is an arbitrary stationary additional control signal, generated from a linear combination of measurements up to time t. Any such signal can be expressed as

$$m(t) = G_v v(t) + G_e e(t)$$

where G_v and G_e are stable rational functions. (In practice, $m(t)$ would be generated by utilizing the measurable signals $w(t)$ and $y(t)$. The rational functions G_v and G_e would then include expressions describing the suitably modified, but still stable, closed-loop system. This does not alter the reasoning.) It will be demonstrated that $m(t) = 0$ is the optimal choice. Use (2.4) on the system. The cost function is then

$$J = J_1 + 2J_2 + J_3 \tag{A 4}$$

where J_1, the cost with $m(t)=0$, is given by (A 3). Since $v(t)$ and $e(t)$ are mutually uncorrelated and all signals involved are stationary,

$$J_2 = Ey_v(t)q^{-k}\frac{B}{\alpha}G_v v(t) + \rho Ez_v(t)\frac{WFA}{\alpha}G_v v(t)$$

$$+ Ey_e(t)q^{-k}\frac{B}{\alpha}G_e e(t) + \rho Ez_e(t)\frac{WFA}{\alpha}G_e e(t)$$

$$J_3 = E\left(\frac{B}{\alpha}m(t)\right)^2 + \rho E\left(\frac{WFA}{\alpha}m(t)\right)^2$$

It it can be shown that $J_2=0$, it follows that $m(t)=0$ is optimal, since J_1 is unaffected by $m(t)$ and J_3 is non-negative.

Using Parseval's formula together with (A 3), the term J_2 can be expressed as

$$J_2 = \frac{\lambda_v}{2\pi i}\oint_{|z|=1}\left[\frac{Mz^kB_* - \rho WFUW_*F_*A_*}{\alpha H\alpha_*}\right]G_{v*}\frac{dz}{z}$$
$$+\frac{\lambda_e}{2\pi i}\oint_{|z|=1}\left[\frac{RC}{\alpha}\frac{z^kB_*}{\alpha_*} - \rho\frac{WSC}{\alpha}\frac{W_*F_*A_*}{\alpha_*}\right]G_{e*}\frac{dz}{z}$$

Use of the expressions for M and U and of the spectral factorization (3.1) gives

$$J_2 = \frac{\lambda_v}{2\pi i}\oint\frac{(z^{-d+k}DFGRB_* - \rho z^{-d}DFGWW_*F_*A_*S - r\beta\beta_*Q)}{H\alpha\alpha_*}G_{v*}\frac{dz}{z}$$
$$+\frac{\lambda_e}{2\pi i}\oint\frac{C(z^kRB_* - \rho WW_*F_*A_*S)}{\alpha\alpha_*}G_{e*}\frac{dz}{z} \qquad \text{(A 5)}$$

Multiplication of (3.2) by S and of (3.3) by R followed by subtraction gives

$$z\alpha X_* = C(z^kRB_* - \rho WW_*F_*A_*S) \qquad \text{(A 6)}$$

Multiply (3.4) by β and express $z\beta X_*$ with (A 6), using $\alpha=\beta C$. This gives

$$z^{-d+1}DFG\beta X_* = r\beta\beta_*Q + z\beta CHL_*$$

$$z^{-d}DFG(z^kRB_* - \rho WW_*F_*A_*S) - r\beta\beta_*Q = z\alpha HL_* \qquad \text{(A 7)}$$

The use of (A 7) in the first term of (A 5) and of (A 6) in the second term gives

$$J_2 = \frac{\lambda_v}{2\pi i}\oint\frac{z\alpha HL_*}{H\alpha\alpha_*}G_{v*}\frac{dz}{z} + \frac{\lambda_e}{2\pi i}\oint\frac{z\alpha X_*}{\alpha\alpha_*}G_{e*}\frac{dz}{z} = 0$$

This expression is zero since, after cancelling $z\alpha H$, the integrands have no poles inside the integration path; stability of $\alpha(z^{-1})$ implies that $\alpha_*(z)$ has zeros outside $|z|=1$ only. (We see that $L_*(z)$ and $X_*(z)$ must have only non-negative powers of z as arguments. Negative powers of z would introduce poles at the origin, and J_2 would not vanish.) Thus, with $J=J_1+J_3$, the choice $m(t)=0$ is optimal. □

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