Passivity-based Control for Multi-Vehicle Systems Subject to String Constraints

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Abstract

In this paper, we show how heterogeneous bidirectional vehicle strings can be modelled as port-Hamiltonian systems. Analysis of stability and string stability within this framework is straightforward and leads to a better understanding of the underlying problem. Nonlinear local control and additional integral action is introduced to design a suitable control law guaranteeing l_2 string stability of the system with respect to bounded disturbances.

Key words: Hamiltonian Systems, String Stability, Multi-Vehicle Systems

1 Introduction

In the field of coordinated systems, formation control is one of many control objectives. A group of N vehicles (e. g. platoon or string) is required to follow a given reference trajectory while the vehicles keep a prescribed distance to neighbouring vehicles. In its simplest form the vehicles in the platoon are considered to move in a single straight line. While many different solutions to this problem have been proposed, most researchers consider a decentralised control structure where each vehicle in the string uses a local controller with locally available measurements instead of a global, centralised controller.

In most cases it is straightforward to design a local controller to achieve stability of a string in the Lyapunov sense. However, it is well known that error signals can amplify when travelling through the string resulting in growth of the local error norm with the position in the string. This effect is referred to as *string instability*, e.g. in [10, 14, 17], or *slinky effect*, e.g. in [3, 6, 18, 21]. A well known definition of l_p string stability has been proposed in [19].

Different approaches have been proposed to guarantee string stability of unidirectional vehicle strings, where each local controller only considers information relating to a group of vehicles in front. These approaches include: (i) introducing a time headway, [3]; (ii) local controllers that depend on the position within the string, [8]; (iii) using the velocity or the acceleration of the lead vehicle; or, (iv) propagating the reference velocity to each vehicle within the platoon, e. g. [10, 20] or [2], respectively.

This paper studies heterogenous, bidirectional, nonlinear strings of vehicles. ("Heterogeneous" refers to the fact that the dynamics of each vehicle and local controller might differ between the vehicles; and bidirectional means that information propagates both upstream and downstream in the string.) It was shown in [1, 17] that similar to unidirectional strings, linear, symmetric bidirectional strings with two integrators in the open loop and constant spacing are always string unstable. The definition used in [1, 17] – even though often not explicitly mentioned – requires that the L_2 norm of the local error vector, i. e. $||e(\cdot)||_2 = \sqrt{\int_0^\infty \sum_{k=0}^N |e_k(t)|^2 \mathrm{d}t}$ is bounded for all disturbances d in L_2.

Different approaches to overcome this limitation have been proposed. It is shown in [10] that this type of string stability can be guaranteed given a sufficiently large coupling

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with the leader position. In [2] a linear, bidirectional string of N vehicles is approximated as a PDE. It is shown that the least stable eigenvalue of the PDE approaches the origin with $O(1/N^2)$ if the string is symmetric and $O(1/N)$ if the string is asymmetric. However, knowledge of the steady state / reference velocity is needed. Both proposed solutions depend on either perfect communication between the leading vehicle and the rest of the string or perfect knowledge of the reference signal. Further, any reference signal changes have to be communicated to every vehicle in the string. In case sufficiently fast and lossless communication of the necessary information cannot be guaranteed, it might not be possible to guarantee string stability. Thus, both control structures suffer from a serious risk if the number of vehicles increases. Therefore, it is desirable to find alternatives that do not depend on global communication or global reference knowledge but local measurements and local communication between a small number of direct neighbours.

A different approach to ensure string stability was considered in [5]. Modelling a symmetric bidirectional string as a mass-spring-damper system, it is shown that string stability with constant spacing can be guaranteed if the damping coefficients or the inverse compliances of the springs between the vehicles, respectively, grow with the string length N. This also seems undesirable in practise. Since controller parameters cannot be chosen infinitely high this implies that the string cannot be extended without bound.

It is the aim of this paper to offer an alternative approach to this problem. First, the definition of l_p string stability proposed in [19], although very useful for unidirectional strings, seems uninformative and too restrictive for bidirectional strings. Recall that it has been shown that string stability cannot be achieved for linear, symmetric, bidirectional strings with two integrators in the open loop, constant spacing and without global communication or reference signal knowledge, [1, 17], and the alternatives suffer from undesirbale risks and limitations above. Thus, we first propose a definition of l_p string stability that is less restrictive than the definition given in [19]. This definition proves to be useful in guaranteeing an upper bound for the distances between the vehicles at all times. Details can be found in the following section. Second, a control algorithm is proposed that does not suffer from any of the above mentioned disadvantages. That is, the design does not require any global communication or global knowledge of the reference information. Further, it is possible to choose all control parameters in a defined bound without the need to limit the number of vehicles. We also propose a different method to model such vehicle strings, that is to use port-Hamiltonian system theory, [11]. This approach offers significant advantages over methods known in the string stability literature. The port-Hamiltonian systems description allows direct and easy to follow stability and string stability proofs and thus offer a better insight into the underlying problem. The method also allows an insight on limitations and advantages of similar system designs presented in the literature such as in [2, 5]. Further, both linear and nonlinear systems can be described as port-Hamiltonian systems.

The paper is organised as follows: The problem of interest in this paper (i. e. string stability of port-Hamiltonian systems describing vehicle platoons) is presented in Section 2. After introducing local control to a string of vehicles in Section 3, integral action will be added in Section 4. The paper ends with a numerical example in Section 5 and concluding remarks in Section 6. Some preliminary results have been reported in [9].

2 Notation and Problem Formulation

2.1 Notation

The L_2 vector norm is given by $|x|_2 = |x| = \sqrt{x^{\mathrm{T}} x}$ and the L_2 and L_∞ vector function norms by $\|x(\cdot)\|_2 = \sqrt{\int_0^\infty |x(t)|^2 \mathrm{d}t}$ and $\|x(\cdot)\|_\infty = \sup_{t\geq 0} |x(t)|$, respectively. For a scalar function $H(x)$ of a vector $x = [x_1, x_2, \ldots, x_n]^{\mathrm{T}}$ its gradient is defined as $\nabla H(x) = \left[\frac{\partial H(x)}{\partial x_1}, \frac{\partial H(x)}{\partial x_2}, \ldots, \frac{\partial H(x)}{\partial x_n}\right]^{\mathrm{T}}$. We denote the state, steady state and the disturbance vector by the column vectors (generally denoted col) $x(t) = \mathrm{col}(x_1(t), \ldots, x_N(t))$, $x_0 = \mathrm{col}(x_{1_0}, \ldots, x_{N_0})$ and $d(t) = \mathrm{col}(d_1(t), \ldots, d_N(t))$. The column vector of ones is denoted by $\underline{1}$ and $\vec{e}_i$ is the ith canonical vector of length N. Similarly we denote the diagonal matrix $A \in \mathbb{R}^{N\times N}$ with diagonal entries $a_1, \ldots a_N$ as $A = \mathrm{diag}(a_1, \ldots a_N)$.

2.2 System Description

We consider a system of N vehicles with mass m_i where $i = 1, 2, \cdots, N$ denotes the position within the string. The motion equations of the system can be described using the momentum and position of each vehicle, i.e. p_i and q_i, as follows

$$\dot{p}_i = F_i + d_i \tag{1}$$
$$\dot{q}_i = m_i^{-1} p_i \tag{2}$$

where F_i is the control force on the vehicle, d_i is the disturbance, and the momentum satisfies $p_i = m_i v_i$, where v_i is the velocity. The local position error between the ith vehicle and its direct predecessor is denoted

$$\Delta_i = q_{i-1} + l_i - q_i. \tag{3}$$

Note that l_i is the desired safety distance between the vehicles plus the length of vehicle $i-1$ or i depending on whether the position of the front or the rear of each vehicle i is used as position q_i. As a minimal distance is usually required between vehicles for obvious safety reasons, we assume $l_i > 0$ for all i. The position q_0 is the reference position available to the first vehicle in the string. The dynamics of the string system described in momenta of the vehicles and separation distance between the vehicles can be described by

$$\begin{bmatrix} \dot{p} \\ \dot{\Delta} \end{bmatrix} = \begin{bmatrix} 0 & S^{\mathrm{T}} \\ -S & 0 \end{bmatrix} \nabla H(p,\Delta) + \begin{bmatrix} F \\ 0 \end{bmatrix} + \begin{bmatrix} d \\ \vec{e}_1 v_0 \end{bmatrix}, \tag{4}$$

where $\Delta, p \in \mathbb{R}^N$ are the displacement and momentum vectors, i. e. $\Delta = \mathrm{col}(\Delta_1,\ldots,\Delta_N)$, $p = \mathrm{col}(p_1,\ldots,p_N)$, and the control force vector is $F = \mathrm{col}(F_1,\ldots,F_N)$. The function $H(p,\Delta)$ is given by

$$H(p,\Delta) = \frac{1}{2} p^{\mathrm{T}} M^{-1} p. \tag{5}$$

The matrix $M \in \mathbb{R}^{N\times N}$ is the constant and positive definite inertia matrix $M = \mathrm{diag}(m_1,\ldots,m_N)$. The matrix S has the bidiagonal form

$$S = \begin{bmatrix} 1 & 0 & \cdots & \cdots & 0 \\ -1 & 1 & \ddots & & \vdots \\ 0 & -1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 & 0 \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix}. \tag{6}$$

2.3 Control Objectives

The local control objective for each vehicle is to bring its local error to zero using local (distributed) control and only locally available data. The control force F_i will be chosen such that only data from a group of neighbours of the ith vehicle (both preceding and following vehicles) are needed. The controller for the first vehicle in the string aims to follow a given trajectory q_0 and also minimise the local position error towards a group of following vehicles. In the simplest setting the reference signal is considered to be a ramp with constant velocity v_0, i.e. $q_0 = v_0 t$. Note that the vehicles within the string (apart from a limited group at the beginning of the string) do not have access to the reference signal and therefore have to adjust their position and momentum indirectly by forcing their local position error to zero.

The overall control objective is to achieve "string stability" or "scalability", that is, the norm of the local states of the complete string do not grow without bound as N increases for nonzero disturbances or initial conditions. Considering the general definition of l_p string stability in [19] and [15], we use the following definition of l_2 string stability with respect to disturbances:

Definition 1 *Consider a system described by $\dot{x} = g(x,d)$ with states $x \in \mathbb{R}^N$, $g \in \mathbb{R}^N$ satisfying $g(x^*,0) = 0$ and disturbances d. The equilibrium x^* is l_2 string stable with respect to disturbances $d(t)$, if given any $\epsilon > 0$, there exists $\delta_1(\epsilon) > 0$ and $\delta_2(\epsilon) > 0$ (independent of N) such that*

$$|x(0) - x^*| < \delta_1(\epsilon) \quad \textit{and} \quad \|d(\cdot)\|_2 < \delta_2(\epsilon) \tag{7}$$

implies

$$\|x(t) - x^*\|_\infty = \sup_{t\geq 0} |x(t) - x^*| < \epsilon \quad \forall N \geq 1. \tag{8}$$

3 Local Control

In this section, a local distributed controller for a bidirectional vehicle string system is designed. The local control structure is motivated by previous results in the field of mechanical engineering. When choosing control actions between the vehicles that can be modelled as virtual springs and dampers between the vehicles, the overall system can easily be written as a port-Hamiltonian system.

The control forces consist of the "spring forces" F_i^{s}, that depend on the position errors Δ_i, the "damper forces" F_i^{r}, that depend on the velocity differences between two neighbouring vehicles, and the "drag forces" F_i^{d} describing the friction of the vehicles towards the ground. Assume the spring force between vehicle $i-1$ and i is given by the function $f_i^{\mathrm{s}}(\Delta_i)$, which is locally Lipschitz for all Δ_i within the domain of definition, strictly monotonic and satisfies $f_i^{\mathrm{s}}(\Delta_i)\Delta_i \geq 0$ and $f_i^{\mathrm{s}}(\Delta_i) = 0$ only for $\Delta_i = 0$ for all $i = 1,\ldots,N$. Thus, $F^{\mathrm{s}} = S^{\mathrm{T}} f^{\mathrm{s}}(\Delta)$ where $f^{\mathrm{s}}(\Delta)$ is the column vector with entries $f_i^{\mathrm{s}}(\Delta_i)$. Also assume that the condomian or target set of f_i^{s} covers the complete real axis between $-\infty$ and ∞. Thus, the function f_i^{s} is invertible and $f_i^{\mathrm{s}^{-1}}$ is its inverse. The vector of inverses is denoted by $f^{\mathrm{s}^{-1}}$.

Remark 2 *Note that nonlinear springs yield some significant advantages over linear spring models: First, one could use barrier functions on the potential energy to prevent collisions between the vehicles in the platoon. To do this, the stiffness of the springs have to increase when the distances between vehicles decrease. For instance a good choice is to design the springs such that $f_i^{\mathrm{s}}(\Delta_i) \to -\infty$ as $\Delta_i \to -l_i$. Thus, in case the spatial error between the desired distance and the actual distance between the vehicles approaches $-l_i$ which corresponds to the distance between the vehicles approaching 0, the spring gets infinitely stiff to prevent a crash. Second, contrary to the linear controllers, where the control action is proportional to the error, nonlinear controller allows a large variety of possibilities to achieve the specified behaviour. For example, nonlinear springs allow to bound the control input when the position error is large, which is impossible using linear controllers. Note that in case f_i^s is not defined for all Δ_i, for instance for barrier functions, the stability analysis should ensure that the initial conditions are chosen such that $f_i^{\mathrm{s}}(\Delta_i(0))$ exists and that the trajectories of the system remain within the set of interest.*

Assuming linear damping forces of the form $F_i^{\mathrm{r}} =$

$R_i(m_{i-1}^{-1}p_{i-1} - m_i^{-1}p_i)$ and linear drag forces of the form $F_i^{\mathrm{d}} = b_i m_i^{-1} p_i$, the overall control force can be described as

$$F = -(B+R)M^{-1}p + \vec{e}_1 R_1 v_0 + S^{\mathrm{T}} f^{\mathrm{s}}(\Delta) \tag{9}$$

with the constant matrices

$$R = \begin{bmatrix} R_1+R_2 & -R_2 & 0 & \cdots & 0 \\ -R_2 & R_2+R_3 & -R_3 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & & R_{N-1}+R_N & -R_N \\ 0 & \cdots & 0 & -R_N & R_N \end{bmatrix} \tag{10}$$

and $B = \mathrm{diag}(b_1, \ldots, b_N)$ where the entries of matrices B and R are design parameters of the controller and $0 < R_i, b_i < \infty$ for all i.

We will show that the system is asymptotically stable with respect to the equilibrium (p^*, Δ^*). However, the values of the displacements in steady state are undesirable and grow with the string length N in presence of a nonzero reference velocity v_0.

Lemma 3 *Consider the string system* (4) *in closed loop with the control law* (9) *and where the initial spatial deviation* $\Delta_i(0)$*, the equilibrium state* Δ_i^* *and all values of* Δ_i *in between are in the domain of definition of* $f_i^{\mathrm{s}}(\Delta_i)$ *for all i. Consider further that* $f_i^{\mathrm{s}}(\Delta_i)$ *is a strictly monotonically increasing function satisfying* $f_i^{\mathrm{s}}(0) = 0$*. Then, the equilibrium* $(p^*, \Delta^*) = \left(M\underline{1}v_0, f^{\mathrm{s}^{-1}}\left(S^{-\mathrm{T}}B\underline{1}v_0\right)\right)$ *is globally asymptotically stable in the absence of disturbances, i.e.* $d = 0$*.*

PROOF. Given (9) and (4) the closed loop has the port-Hamiltonian form

$$\begin{bmatrix} \dot{p} \\ \dot{\Delta} \end{bmatrix} = \begin{bmatrix} -(B+R) & S^{\mathrm{T}} \\ -S & 0 \end{bmatrix} \nabla H_{\mathrm{cl}}(p, \Delta) \tag{11}$$

with the closed-loop Hamiltonian function

$$H_{\mathrm{cl}}(p,\Delta) = \frac{1}{2}(p - M\underline{1}v_0)^{\mathrm{T}} M^{-1}(p - M\underline{1}v_0) + \sum_{i=1}^{N} \int_{\Delta_i^*}^{\Delta_i} \left(f_i^{\mathrm{s}}(w) - \sum_{k=i}^{N} b_k v_0 \right) \mathrm{d}w \tag{12}$$

where $\sum_{k=i}^{N} b_k v_0$ is the ith entry of the vector $S^{-\mathrm{T}}B\underline{1}v_0$. Note that the second right hand term in (12) is non-negative. The nonlinear spring functions $f_i^{\mathrm{s}}(\Delta_i)$ are strictly monotonically increasing and satisfy $f_i^{\mathrm{s}}(0) = 0$. Also note that $f^s(\Delta^*) = S^{-T}B1v_0$. Adding the term $-\sum_{k=i}^{N} b_k v_0$ ensures that the integrand is positive for $\Delta_i > \Delta_i^*$ which leads to the integral being positive. In case $\Delta_i < \Delta_i^*$ the integrand is negative, which leads to the integral from Δ^* to Δ being also positive. Hence, the integral is non-negative and only equal to zero for $\Delta_i = \Delta_i^*$.

Using H_{cl} as a Lyapunov function candidate and computing its time derivatives yields

$$\begin{aligned} \dot{H}_{\mathrm{cl}}(p,\Delta) &= \nabla^{\mathrm{T}} H_{\mathrm{cl}}(p,\Delta) \begin{bmatrix} -(B+R) & S^{\mathrm{T}} \\ -S & 0 \end{bmatrix} \nabla H_{\mathrm{cl}}(p,\Delta) \\ &= -\nabla_p^{\mathrm{T}} H_{\mathrm{cl}}(p,\Delta)(B+R)\nabla_p H_{\mathrm{cl}}(p,\Delta) \\ &\leq 0 \end{aligned} \tag{13}$$

This implies Lyapunov stability. Note also that this implies that the spatial deviation Δ_i remains in the definition set of $f_i^{\mathrm{s}}(\Delta_i)$ if the initial spatial deviation $\Delta_i(0)$, the equilibrium state Δ_i^* and all values of Δ_i in between are in the domain of definition of $f_i^{\mathrm{s}}(\Delta_i)$ for all i. Asymptotic stability follows by applying the Invariance principle, which ensures that the trajectories of the states converge to the largest invariant set $\mathcal{S}$, i.e. $\lim_{t\to\infty} p(t) = p^*$ and $\lim_{t\to\infty} \Delta(t) = \Delta^*$ such that $H_{\mathrm{cl}}(p^*, \Delta^*) = 0$, (see e.g. [7]). □

Note that $\Delta_1^* = f_1^{\mathrm{s}^{-1}}\left(\sum_{k=1}^{N} b_k v_0\right)$. Thus, for positive parameters b_i the steady state values of the spatial separation between the vehicles is non-zero. Moreover, if a positive lower bound on the drag and compliance coefficients exists, i. e. $\min_i b_i > \underline{b} > 0$ and $\min_i c_i > \underline{c} > 0$, the argument of $f_1^{\mathrm{s}^{-1}}$ grows with N. Depending on the form of f_1^{s}, Δ^* might grow without bound at the beginning of the string. In any case, Δ^* is not a desired equilibrium point. This effect could be avoided by choosing parameters b_i that decrease sufficiently fast with i. Note that there are examples in the literature, where string stability can be guaranteed if control parameters grow with the position. (Such as growing R_i in [5].) However, this implies that such a vehicle string might not be scalable in practice and is therefore undesirable. Another option is to assume that each agent has perfect knowledge of the reference signal (in its simplest case the reference velocity as discussed in [2]) and to use this knowledge to compensate the influence of the drag onto the steady state. However, this requires the communication of any changes in the reference signal to all agents.

4 Integral Action

Studying a bidirectional vehicle string in the previous section showed that introducing local control based on a mass-spring-damper system can guarantee stability of an equilibrium point which is not the desired equilibrium. Thus, integral action will be introduced in this section to ensure string stability of the desired equilibrium. Also, it will be shown that using suitable integral action allows to reject constant unknown disturbances. The following theorem studies such a controller with additional integral action.

Theorem 4 *Assume the disturbances d include a constant component d_c and a dynamical component $d_\mathrm{d}(t)$ such that $d = d_\mathrm{c} + d_\mathrm{d}(t)$ and there exists a constant $D < \infty$ satisfying $\|d_\mathrm{d}(\cdot)\|_2 \leq D$. Consider the string system (4) with a reference signal with constant velocity v_0, disturbance d in closed loop with a controller obtained by adding the control in Lemma 3 and the additional dynamic control force F_IA, i. e.*

$$\begin{aligned} F &= -(B+R)M^{-1}p + \vec{e}_1 R_1 v_0 + S^\mathrm{T} f^\mathrm{s}(\Delta) + F_\mathrm{IA} && (14)\\ F_\mathrm{IA} &= -A_\mathrm{p} M^{-1} p + MKS^\mathrm{T} f^\mathrm{s}(\Delta) - (B+R+A_\mathrm{p})Kz_3 && (15)\\ \dot{z}_3 &= -S^\mathrm{T} f^\mathrm{s}(\Delta). && (16) \end{aligned}$$

where A_p is a constant diagonal matrix $A_\mathrm{p} = \mathrm{diag}(a_{\mathrm{p}_1},\ldots,a_{\mathrm{p}_N})$ with $0 \leq a_{\mathrm{p}_i} < \infty$ for all i, $K \in \mathbb{R}^{N\times N}$ is a constant diagonal positive matrix $K = \mathrm{diag}(k_1,\ldots,k_N)$ with $0 < k_i < \infty$ for all i. Then

(1) for $d_\mathrm{d}(t) = 0$ the desired equilibrium point

$$(p^*,\Delta^*,z_3^*) = \left(M\underline{1}v_0, 0, K^{-1}(B+R+A_\mathrm{p})^{-1}\left(d_\mathrm{c} - \left(B+A_\mathrm{p}\right)\underline{1}v_0\right)\right) \quad (17)$$

is globally asymptotically stable (despite the presence of constant unknown disturbances d_c),

(2) the system is passive with input d_d, output $y = \nabla_{z_1} H_\mathrm{z}(z)$ and storage function $H_\mathrm{z}(z)$ and constant disturbances d_c are rejected, and

(3) the system is l_2 string stable with respect to the dynamic disturbances $d_\mathrm{d}(t)$.

PROOF. (1): Consider the following change of coordinates

$$\begin{aligned} z_1 &= p - M\underline{1}v_0 + MK(z_3 - \alpha), && (18)\\ z_2 &= \Delta && (19) \end{aligned}$$

with $\alpha = K^{-1}(B+R+A_\mathrm{p})^{-1}\left(d_\mathrm{c} - \left(B+A_\mathrm{p}\right)\underline{1}v_0\right)$. Hence,

$$\begin{aligned} \dot{z}_1 =& S^\mathrm{T} f^\mathrm{s}(\Delta) - RM^{-1}p + \vec{e}_1 R_1 v_0 - BM^{-1}p + d - A_\mathrm{p}M^{-1}p\\ &+ MKS^\mathrm{T} f^\mathrm{s}(\Delta) - (B+R+A_\mathrm{p})Kz_3 - MKS^\mathrm{T} f^\mathrm{s}(\Delta)\\ =& -(B+R+A_\mathrm{p})M^{-1}z_1 + S^\mathrm{T} f^\mathrm{s}(\Delta) \end{aligned} \quad (20)$$

and

$$\dot{z}_2 = -SM^{-1}(p - M\underline{1}v_0) = -SM^{-1}z_1 + SK(z_3 - \alpha).$$

Thus, the closed loop dynamics have the port Hamiltonian form

$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\\ \dot{z}_3\end{bmatrix} = \begin{bmatrix} -(B+R+A_\mathrm{p}) & S^\mathrm{T} & 0\\ -S & 0 & S\\ 0 & -S^\mathrm{T} & 0\end{bmatrix} \nabla H_\mathrm{z}(z) \quad (21)$$

with the Hamiltonian function

$$\begin{aligned} H_\mathrm{z}(z_1,z_2,z_3) =& \frac{1}{2} z_1^\mathrm{T} M^{-1} z_1 + \sum_{i=1}^{N} \int_0^{z_{2_i}} f_i^\mathrm{s}(w)\mathrm{d}w\\ &+ \frac{1}{2}(z_3-\alpha)^\mathrm{T} K (z_3 - \alpha). \end{aligned} \quad (22)$$

Note that it can be shown in a similar way as below (12) that $H_\mathrm{z}(z)$ is anon-negative function. As the spring functions f_i^s are strictly monotonically increasing and satisfy $f_i^\mathrm{s}(0) = 0$ the integrand of the second right hand term in (22) is positive for $z_{2_i} > 0$ and negative for $z_{2_i} > 0$. Hence, the integral is non-negative and only equal to zero for $z_{2_i} = 0$. It can be shown in a similar way as in the proof of Lemma 3 that the system is asymptotically stable by using the Hamiltonian as a Lyapunov function and considering that $B + R + A_\mathrm{p}$ is positive definite.

(2): Note that with constant disturbances d_c and additional dynamic disturbances $d_\mathrm{d}(t)$ the system description changes to

$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\\ \dot{z}_3\end{bmatrix} = \begin{bmatrix} -(B+R+A_\mathrm{p}) & S^\mathrm{T} & 0\\ -S & 0 & S\\ 0 & -S^\mathrm{T} & 0\end{bmatrix} \nabla H_\mathrm{z}(z) + \begin{pmatrix} d_\mathrm{d}(t)\\ 0\\ 0\end{pmatrix}. \quad (23)$$

with $H_\mathrm{z}(z)$ given in (22). Using $H_\mathrm{z}(z)$ as a Lyapunov function candidate, taking its derivative and setting $y = \nabla_{z_1} H_\mathrm{z}(z)$ yields

$$\dot{H}_\mathrm{z}(z) \leq -\lambda_\mathrm{min}(B+R+A_\mathrm{p})|y|^2 + y^\mathrm{T} d_\mathrm{d}(t). \quad (24)$$

As $B+R+A_\mathrm{p}$ is positive definite, the system is passive, [12].

(3): Extending the argument in (24) leads to

$$\begin{aligned} \dot{H}_\mathrm{z}(z) \leq& -\frac{\lambda_\mathrm{min}(B+R+A_\mathrm{p})}{2}|y|^2 + \frac{1}{2\lambda_\mathrm{min}(B+R+A_\mathrm{p})}|d_\mathrm{d}(t)|^2\\ &- \frac{\lambda_\mathrm{min}(B+R+A_\mathrm{p})}{2}\left| y - \frac{1}{\lambda_\mathrm{min}(B+R+A_\mathrm{p})} d_\mathrm{d}(t)\right|^2\\ \leq& \frac{1}{2\lambda_\mathrm{min}(B+R+A_\mathrm{p})}|d_\mathrm{d}(t)|^2. \end{aligned} \quad (25)$$

Hence,

$$H_\mathrm{z}(z(t)) \leq H_\mathrm{z}(z(0)) + \frac{1}{2\lambda_\mathrm{min}(B+R+A_\mathrm{p})}\|d_\mathrm{d}(\cdot)\|^2. \quad (26)$$

Note that since R is positive semidefinite and B is positive definite, it can be shown using Gershgorin's Theorem that $\lambda_{\min}(B + R + A_{\rm p}) \geq \min_i(b_i + a_{{\rm p}_i})$. Thus,

$$\begin{aligned} H_{\rm z}(z(t)) \leq& \left(2\min_i(m_i)\right)^{-1}|z_1(0)|^2 + \sum_{i=1}^{N}\int_0^{z_{2_i}(0)} f_i^{\rm s}(w){\rm d}w \\ &+ \frac{\max_i k_i}{2}|z_3(0)-\alpha|^2 + \left(2\min_i\left(b_i + a_{{\rm p}_i}\right)\right)^{-1}\|d(\cdot)\|^2 \\ \leq& \left(2\min_i(m_i)\right)^{-1}|z_1(0)|^2 + \sum_{i=1}^{N} L_i|z_{2_i}(0)|^2 \\ &+ \frac{\max_i k_i}{2}|z_3(0)-\alpha|^2 + \left(2\min_i\left(b_i + a_{{\rm p}_i}\right)\right)^{-1}\|d(\cdot)\|^2 \end{aligned} \quad (27)$$

where L_i is the Lipschitz constant for $f_i^{\rm s}(w)$ for $w \in [0, z_{2i}(0)]$. (Since $f_i^{\rm s}$ is monotonically increasing, it is equivalent to require that for every $w = \overline{w}$ there exists a constant c such that $f_i^{\rm s}(\overline{w}) = c\overline{w}$.) Since the mass m_i, the drag coefficient b_i and the integral action control parameters $a_{{\rm p}_i}$ and k_i for each vehicle are positive, $H_{\rm z}(z)$ is bounded for all N if $|z(0)|$ and $\|d(\cdot)\|_2$ do not increase with N. Given the terms including z_1 and z_3 in (22) are quadratic and $f_i^{\rm s}$ is strictly monotonically increasing, an upper bound on $H_{\rm z}(z)$ implies that the states z are also bounded. Further, note that $\Delta = z_2$ and p is a linear combination of z_1 and z_3 plus two constant offsets terms. Hence, p and Δ are also bounded if an upper bound for $H_{\rm z}(z)$ exists. Therefore, the system is string stable. □

Note that the integral action introduces more design parameters in the controller by adding the matrices A_p and K in the control law.

Remark 5 *The procedure to design the integral action follows ideas proposed in [4,13,16]. The change of coordinates* (18)-(19) *is fundamental for this design. The procedure is as follows: First, extend the state vector by adding new states* z_3*, and choose their dynamics to be driven by the displacements* Δ*. This will result in integral action on the displacements. Second, choose the change of coordinates* (19) *to preserve the variables on which the additional integral action is required. Third, compute the derivative with respect to time of* (19)*, and replace the derivative of the states by their state equations from the open loop system and desired closed loop. Then solve this equation for* z_1 *to obtain the change of coordinate* (18)*. Finally, compute the time derivative of* (18)*, replace the derivative of the state by their corresponding state equations, and solve the resulting equation for the control law* F_{IA}*. The block-matrices and functions of the closed loop* (21) *have to be chosen to satisfy the limited information available in each vehicle, to guarantee string stability and to ensure that the control law does not depend on the unknown disturbance.*

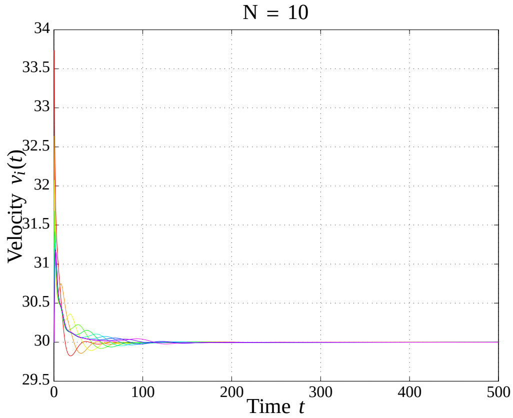

Figure 1. Velocities v_i over time for a string of 10 vehicles for $i = 1$ (red) ... $i = 10$ (purple)

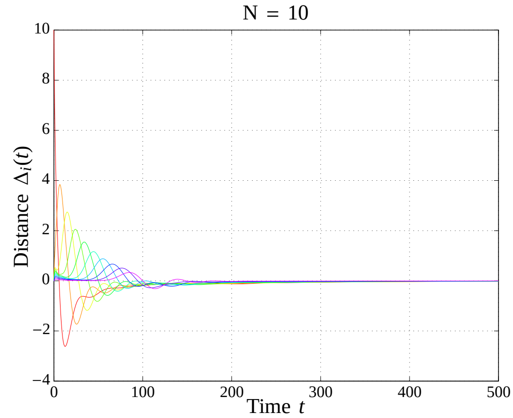

Figure 2. Displacements Δ_i over time for a string of 10 vehicles for $i = 1$ (red) ... $i = 10$ (purple)

5 Example

Ten homogeneous strings of length $N = 10, 20, \ldots, 100$ with local control and integral action control ($m_i = 1$, $f_i^{\rm s}(\Delta_i) = \Delta_i + 0.1\Delta_i^2$, $b_i = 0.1$, $r_i = 20$ and $K_i = 100$ for all $i = 1, 2, \ldots, N$) have been simulated. Nonzero initial conditions for p and Δ have been used for the first vehicle in the string. Figures 1-4 show the evolution of the velocities and inter-vehicle distances for a string of 10 and 100 vehicles. It can be observed that the disturbance is not amplified when traveling through the longer string. Instead, all deviations from the steady state values remain bounded independently of the string size and the position within the string.

In a second set of simulations, ten heterogeneous strings of length $N = 10, 20, \ldots, 100$ with local control and integral action control have been simulated. The parameters were chosen randomly in the ranges $m_i \in [0.8, 1.2]$, $f_i^{\rm s}(\Delta_i) = c_i\Delta_i +$

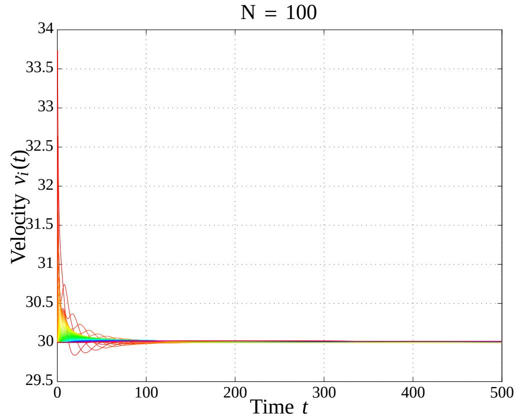

Figure 3. Velocities v_i over time for a string of 100 vehicles for $i = 1$ (red) ... $i = 100$ (purple)

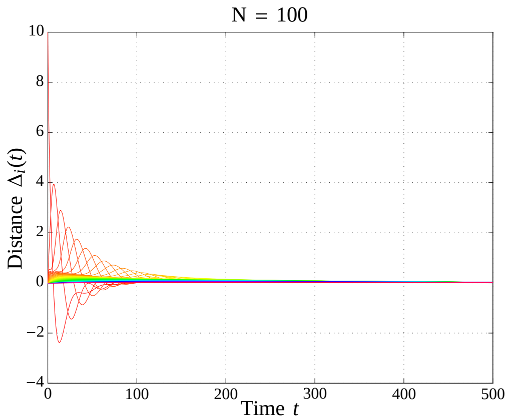

Figure 4. Displacements Δ_i over time for a string of 100 vehicles for $i = 1$ (red) ... $i = 100$ (purple)

$0.1\Delta_i^2$ with $c_i \in [0.8,1.2]$, $b_i \in [0.08,0.12]$, $r_i \in [18,22]$ for all $i = 1,2,\ldots,N$. The maximal point wise norm of the complete state vector $z(t)$, i.e. $\max_t |z(t)|$, for all string sizes is shown for the homogeneous strings in Figure 5 and for the heterogeneous strings in Figure 6. One can observe that the maximal deviation from the steady state value does not grow with string size but instead settles on a constant value for $N \geq 40$ for the set of homogeneous strings. As the system parameters of the heterogeneous strings are chosen randomly in a range around the nominal value used in the homogeneous setting, the values of $\max_t |z(t)|$ differ and do not settle on a constant value for sufficiently long strings. However, it can be observed in Figure 6 that despite the variation in $\max_t |z(t)|$, the values are around the same value as for the set of homogeneous strings and remain bounded independently of the string size N.

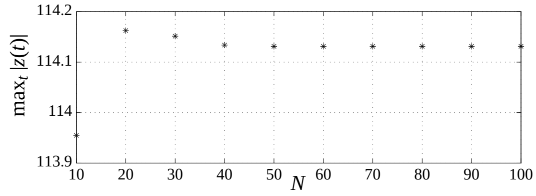

Figure 5. Velocities v_i over time for a string of 100 vehicles

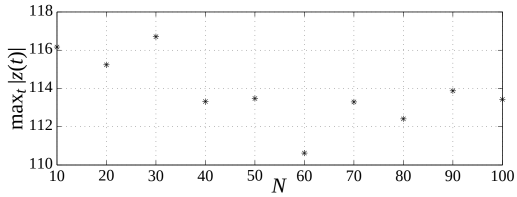

Figure 6. Displacements Δ_i over time for a string of 100 vehicles

6 Conclusions

Modelling bidirectional, heterogeneous vehicle strings as port-Hamiltonian systems offers significant advantages over traditional system descriptions involving ordinary differential equations. It was shown that this system description not only incorporates both linear and nonlinear systems, but also leads to an easy to follow and straightforward stability and string stability analysis.

The local control law proposed in this paper consists of virtual nonlinear springs and dampers between the vehicles and drag towards ground. Suitable integral action control was added to guarantee string stability. The advantage of this approach is that it only relies on decentralised control and locally measurable data of a set of direct neighbours. No global communication with the leading vehicle or global knowledge of the reference signal are necessary and all control parameters can be chosen in defined bounds independent of the string size.

It has been shown in [17] that the common strict form of string stability (requiring the l_2 of all states to be bounded for any l_2 bounded disturbance) cannot be achieved for symmetric homogeneous bidirectional strings with tight spacing and two poles in the open loop of each vehicle in the string. Thus, a different more informative definition has been used. The definition proves to be useful to guarantee point wise (in time) bounded states in bidirectional vehicle strings.

So far only nonlinear springs have been investigated in this work. A more detailed analysis is necessary to study nonlinear dampers, drag or more general nonlinear systems. At the current stage the port-Hamiltonian description based stability analysis is only suitable to cover symmetric communication settings. If the virtual springs and dampers between two agents are modified to allow for unbalanced forces at their ends, an extension of the existing approach is needed.

The local data such as the distances and velocity differences towards neighbouring vehicles are assumed to be available without noise, delay or other real world inaccuracies. A more detailed analysis is needed to investigate the effects of communication or measurement limitations on string stability.

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