Peak-to-average power ratio reduction in OFDM using cyclically shifted phase sequences

G. Lu, P. Wu and D. Aronsson

Abstract: Two approaches for reducing peak-to-average power ratio (PAPR) in orthogonal frequency division multiplexing (OFDM) are proposed that are relied on a set of cyclically shifted phase sequences (CSPS) and implemented using the time domain circular convolution. After multiplying CSPS with the frequency domain data, the signal candidates can be expressed as weighted sum of the circularly shifted OFDM time domain data in the first method, which is called CSPS method. In the second method, weighted coefficients for generating the signal candidates in CSPS method are optimally selected to improve its performance; thus, the second method is referred to as optimised CSPS (OCSPS) method. The performances of the CSPS and OCSPS methods are evaluated using simulated data and compared with those of selective mapping (SLM) and partial transmit sequences (PTS). The simulation results show that both the CSPS and OCSPS methods can reduce the PAPR effectively, and that the OCSPS performs even better than the CSPS. The OCSPS can achieve the same performance as compared to the PTS. A distinct feature of the proposed methods is that only one inverse discrete Fourier transform is needed, and thus, the candidates can be calculated in time domain directly.

1 Introduction

Orthogonal frequency division multiplexing (OFDM) has a high tolerance to frequency selective channels and is spectrally efficient, making it a good candidate for future wireless communication systems [1–3]. In the wireless IP Project [4], for example, an adaptive OFDM radio interface was used in both downlink and uplink in a packet switching wireless cellular system with wide area coverage and high throughput.

However, OFDM has a significant drawback that the transmitted signal has high peak-to-average power ratio (PAPR), which requires very linear, large dynamic range amplifiers that are inefficient and expensive to operate. This is especially used in the power limited scenarios, such as in satellite mobile communication and in the uplink of wireless communication. To avoid the saturation of amplifiers and the consequent in-band distortion and out-of-band radiation, PAPR should be reduced before the signal is fed to the amplifiers. Therefore the reduction of PAPR has attracted great attention of researchers in the wireless communications society and many solutions to PAPR reduction have been proposed. Among many solutions, the distortionless methods are very attractive, since the information in transmitted signals is undistorted. The partial transmit sequences (PTS) [5] and the selective mapping (SLM) [6] are two of the typical distortionless methods. The key of PTS is the optimal combination of phase-rotated signal sub-blocks to minimise the peak power, while in the SLM, the frequency domain data is multiplied by a set of statistically independent sequences and the corresponding time domain signal with the smallest PAPR is selected and transmitted. Both approaches provide improved PAPR statistics and need side information (SI) to recover the original OFDM signal at the receiver.

Since PTS and SLM operate in the frequency domain, the number of inverse discrete Fourier transform (IDFT) operations needed in the two methods is equal to the number of sub-blocks and the phase sequences, respectively. This leads to high computational costs. For the case of interleaved sub-blocks partition [7], Cookey-Tukey FFT is employed to reduce the computational complexity of PTS. However, the performance of the interleaved sub-blocks partition scheme is worse than other two partitioning schemes, such as adjacent and pseudo-random schemes, because of the dependence of the candidates in interleaved sub-blocks partition scheme [8]. To lower the computational costs in optimising the weighting factors in PTS, binary weighting factors were used in [9] to obtain the sub-optimal performance with reduced computational complexity. In [10], a gradient descent search was performed to find the phase factor with reduced complexity compared to the original PTS and better performance than the algorithm in [9]. Because the performance gain of PTS method results from the large number of candidates bearing the same information, one uses certain transformations, such as conjugate, time reversal, cyclical shift, and so on, on PTSs to obtain more candidates and thus increase the performance of PTS [11, 12].

In this paper, two new PAPR reduction methods are developed based on the time domain circular convolution (TDCC) of the discrete Fourier transform (DFT). A set of

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cyclically shifted phase sequences (CSPS) is designed first. Then, employing the similar idea to that in the SLM, the alternative frequency domain data can be obtained via the elementwise multiplication of the original frequency data with CSPS. Thanks to the CSPS, a set of signal candidates can be obtained directly in the time domain, and the original signal can be easily recovered in the frequency domain at the receiver.

2 Review of SLM method and PTS method

Assume that the transmitted data in the frequency domain for one OFDM symbol in an OFDM system is $\boldsymbol{X} = [X_0, X_1, \ldots, X_{N-1}]$ and the number of the sub-carriers N is a power of 2.

2.1 SLM method

Assume that M_1 statistically independent random phase sequences of length N are

$$\boldsymbol{Q}^{(\mu)} = [Q_0^{(\mu)}, Q_1^{(\mu)}, \ldots, Q_{N-1}^{(\mu)}], \quad \mu = 1, \ldots, M_1$$

where $Q_k^{(\mu)} = \mathrm{e}^{j\phi_k^{(\mu)}}$ $(k = 0, \ldots, N-1)$, with $\phi_k^{(\mu)}$ uniformly distributed on $[0, 2\pi)$. Multiplying $\boldsymbol{X}$ elementwise with $\boldsymbol{Q}^{(\mu)}$ produces M_1 candidates $\boldsymbol{X}^{(\mu)} = \boldsymbol{Q}^{(\mu)}\boldsymbol{X}$ $(\mu = 1, \ldots, M_1)$, and using M_1 N-point IDFTs yields the time domain data.

$$x_n^{(\mu)} = \mathrm{IDFT}\{X_k^{(\mu)}\}, \; n = 0, \ldots, N-1, \; \mu = 1, \ldots, M_1$$

Finally, the sequence with the lowest PAPR among the candidates $\boldsymbol{x}^{(\mu)}$ is selected for transmission.

In SLM, the number of necessary IDFTs is equal to that of the statistically independent random phase sequences. Increasing M_1 results in an increase in performance, and unfortunately, in computational costs too.

2.2 PTS method

In PTS method, the frequency domain data $\boldsymbol{X}$ is first partitioned into a set of, say M_2, disjoint sub-blocks $\{\boldsymbol{X}^{(m)}, m = 1, \ldots, M_2\}$, and each of them has N/M_2 non-zero elements. Secondly, new data $\boldsymbol{X}'$ are generated by weighting the M_2 sub-blocks

$$\boldsymbol{X}' = \sum_{m=1}^{M_2} b_m \boldsymbol{X}^{(m)} \tag{1}$$

where $b_m = \mathrm{e}^{j\phi_m}$ $(m = 1, \ldots, M_2)$ with ϕ_m uniformly distributed on $[0, 2\pi)$ are the weighting factors. Using N-point IDFT, one may have the time domain expression of (1)

$$\boldsymbol{x}' = \sum_{m=1}^{M_2} b_m \boldsymbol{x}^{(m)} \tag{2}$$

where $\boldsymbol{x}^{(m)}$ $(m = 1, \ldots, M_2)$ are the IDFT of $\boldsymbol{X}^{(m)}$ and called the PTSs. Finally, to minimise the PAPR of $\boldsymbol{x}'$, the weighting factors b_m are optimally determined via

$$\{\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_{M_2}\} = \underset{\{b_1, b_2, \ldots, b_{M_2}\}}{\arg\min} \{\mathrm{PAPR}(\boldsymbol{x}')\} \tag{3}$$

where

$$\mathrm{PAPR}(\boldsymbol{x}') = 10 \log_{10} \frac{\max\{|x_n'|^2\}}{E\{|x_n'|^2\}}$$

is the PAPR in decibel (dB) of $\boldsymbol{x}'$. The data with the smallest PAPR is $\hat{\boldsymbol{x}} = \sum_{m=1}^{M_2} \hat{b}_m \boldsymbol{x}^{(m)}$ and the optimal weighting factors $\hat{b}_m$ should be transmitted as SI.

Clearly, in PTS, the number of performing IDFTs is equal to the number of sub-blocks M_2. The main computations of PTS method in the transmitter include M_2 N-point IDFTs and the optimisation of the weighting factors. As in SLM, increasing the number of IDFTs' operations, the performance of PTS will increase, but so will the computational complexity. Therefore the trade-off between the performance and the complexity needs to be considered when using PTS or SLM for PAPR reduction.

3 PAPR reduction based on circularly shifted phase sequences

In this section, two methods for reducing PAPR are proposed. The first one is called circularly shifted phase sequences (CSPS) method, and the second one optimised circularly shifted phase sequences(OCSPS) method, in which a set of CSPS is first used, and the signal with the smallest PAPR is then selected for transmission. Only one IDFT operation is needed in the proposed methods.

3.1 Circularly shifted phase sequences

Assume that a random phase sequence of length L is $\boldsymbol{P} = [P_0, P_1, \ldots, P_{L-1}]$, where $P_k = \mathrm{e}^{j\varphi_k}$ with φ_k uniformly distributed on $[0, 2\pi)$ and N/L is an integer M, that is $M = N/L$. Then the periodic extension of $\boldsymbol{P}$ with period L is expressed as

$$P((k))_L = \sum_{i=-\infty}^{\infty} P_{k+iL}$$

where $((k))_L$ is k modulo L. Then, the zero-th to $(N-1)$-th elements of $P((k))_L$ are selected to form a phase sequence $\boldsymbol{B}$ of length-N in the following manner

$$B_k = P((k))_L R_k = \sum_{i=-\infty}^{\infty} P_{k+iL} R_k, \; k = 0, \ldots, N-1 \tag{4}$$

where

$$R_k = \begin{cases} 1, & 0 \le k \le N-1 \\ 0, & \text{else} \end{cases}$$

is rectangular sequence of length N. Obviously, B_k $(k = 0, \ldots, N-1)$ consists of M periods of $P((k))_L$ because of $N = ML$.

By circularly shifting B_k by $1, \ldots, L-1$ samples to the left, respectively, one can obtain L different phase sequences as follows

$$B_k^{(l)} = \sum_{i=-\infty}^{\infty} P_{k+l+iL} R_k, \; l = 0, \ldots, L-1, \quad k = 0, \ldots, N-1 \tag{5}$$

which are called CSPS and will be used to establish the new method for PAPR reduction.

3.2 CSPS method for PAPR reduction

As in SLM, L frequency domain sequences $\boldsymbol{X}^{(l)}$ $(l = 0, \ldots, L-1)$ can be obtained by multiplying elementwise the L phase sequences $\boldsymbol{B}^{(l)} = \{B_0^{(l)}, \ldots, B_{N-1}^{(l)}\}$ with $\boldsymbol{X}$, respectively

$$\boldsymbol{X}^{(l)} = \boldsymbol{X} \cdot \boldsymbol{B}^{(l)} = [B_0^{(l)} X_0, B_1^{(l)} X_1, \ldots, B_{N-1}^{(l)} X_{N-1}],$$
$$l = 0, \ldots, L-1 \tag{6}$$

Then the time domain data can be expressed, using the circular convolution property of the DFT, as

$$x_n^{(l)} = \text{IDFT}\{X_k^{(l)}\} = x_n \, \textcircled{N} \, b_n^{(l)}, \quad l = 0, \ldots, L-1, \quad n = 0, \ldots, N-1 \tag{7}$$

where x_n, $b_n^{(l)}$ are the N-point IDFTs of $\boldsymbol{X}$ and $\boldsymbol{B}^{(l)}$, respectively, that is, $x_n = \text{IDFT}\{X_k\}$, $b_n^{(l)} = \text{IDFT}\{B_k^{(l)}\}$, and $\textcircled{N}$ denotes the modulo-N circular convolution. Our aim is to select the signal with the smallest PAPR from the candidates $\boldsymbol{x}^{(l)}$ $(l = 0, \ldots, L-1)$, which is the same as is done in the SLM.

Thanks to the special structure of $\boldsymbol{B}^{(l)}$ in (5), $b_n^{(l)}$ can be explicitly written as [13]

$$\begin{aligned} b_n^{(l)} &= \begin{cases} W_N^{nl} p_{n/M}, & n = 0, M, 2M, \ldots, (L-1)M \\ 0, & \text{else} \end{cases} \\ &= \sum_{i=0}^{L-1} W_N^{iMl} p_i \delta(n - iM) \\ &= \sum_{i=0}^{L-1} W_L^{il} p_i \delta(n - iM), \quad l = 0, \ldots, L-1, \\ & \quad n = 0, \ldots, N-1 \end{aligned} \tag{8}$$

where $p_n = \text{IDFT}\{P_k\}$ is the L-point IDFT of $\boldsymbol{P}$, and $W_N^k = e^{-j(2\pi/N)k}$ is the twiddle factor. Since $b_n^{(l)}$ have only L non-zero elements, then (7) can be rewritten as

$$\begin{aligned} x_n^{(l)} &= \sum_{i=0}^{L-1} W_L^{il} p_i [x_n \, \textcircled{N} \, \delta(n - iM)] \\ &= \sum_{i=0}^{L-1} W_L^{il} p_i \{x((n - iM))_N R_n\}, \\ & \quad l = 0, \ldots, L-1, \quad n = 0, \ldots, N-1 \end{aligned} \tag{9}$$

in which $\{x((n - iM))_N R_n)\}$ is the circular right shift of x_n by iM samples, and the L candidates $x_n^{(l)}$ $(l = 0, \ldots, L-1)$ can be calculated directly in the time domain. Then from the L candidates, the one with the smallest PAPR can be selected for transmission. Since the CSPSs play the essential role in the proposed method, thus, the method is named as CSPS method.

To summarise, let us go through the procedure of the CSPS method:

1. obtain the time domain data x_n with one N-point IDFT, that is $x_n = \text{IDFT}\{X_k\}$;
2. compute the candidates $\boldsymbol{x}^{(l)}$ $(l = 0, \ldots, L-1)$ according to (9) in three steps:

(a) circularly right shift x_n to obtain $\{x((n - iM)_N)R_n\}$, $i = 0, \ldots, L-1$
(b) compute new sequences defined as

$$q_n^{(i)} = p_i \{x((n - iM))_N R_n\}, i = 0, \ldots, L-1 \tag{10a}$$

(c) compute the candidates

$$x_n^{(l)} = \sum_{i=0}^{L-1} W_L^{il} q_n^{(i)}, \quad l = 0, \ldots, L-1 \tag{10b}$$

3. select and then transmit the one, say $\tilde{\boldsymbol{x}}$, with the smallest PAPR.

After x_n is obtained by an N-point IDFT, the L candidate sequences $\boldsymbol{x}^{(l)}$ $(l = 0, \ldots, L-1)$ can be calculated directly in time domain, and the phase sequence $\boldsymbol{P}$ of length L and the index of $\boldsymbol{B}^{(l)}$ resulting in the smallest PAPR should be transmitted as SI to the receiver. The block diagram of the CSPS at the transmitter side is illustrated in Fig. 1 and the computation procedure for one candidate using (9) is shown in Fig. 2.

3.3 OCSPS method

The candidates in (9) can be viewed as the superposition of the circular shifts of x_n. Denoting $d_i = W_N^{il} p_i$ $(i = 0, \ldots, L-1)$, (9) can be rewritten as

$$\bar{x}_n = \sum_{i=0}^{L-1} d_i \{x((n - iM))_N R_n\} \tag{11}$$

It is obvious that, in the CSPS, the weights d_i $(i = 0, \ldots, L-1)$ are fixed once the phase sequence $\boldsymbol{B}^{(l)}$ is generated, and only L candidates can be obtained. To improve the performance of CSPS method, more candidates should be employed and the weights d_i $(i = 0, \ldots, L-1)$ can be selected optimally. Hence, the weights are optimally selected by minimising the PAPR of $\bar{\boldsymbol{x}}$ as follows

$$\boldsymbol{d}_{\text{opt}} = \underset{\{d_0, \cdots, d_{L-1}\}}{\arg\min} \ \text{PAPR}\{\bar{\boldsymbol{x}}\} \tag{12}$$

The optimal phase sequence $\boldsymbol{d}_{\text{opt}}$, which is the SI of this method, gives one candidate that has the smallest PAPR. Once the $\boldsymbol{d}_{\text{opt}}$ is received at the receiver, the phase sequence $\boldsymbol{B}_{\text{opt}}$ can be easily reconstructed from periodically extending the $\boldsymbol{D}_{\text{opt}} = \text{DFT}\{\boldsymbol{d}_{\text{opt}}\}$ as in (4). Then the original OFDM

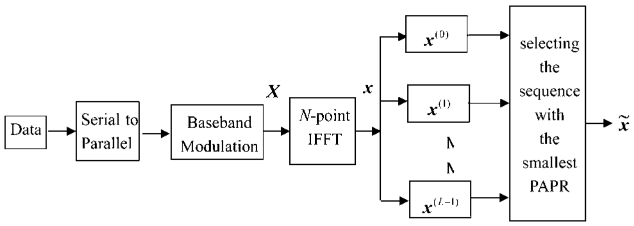

Fig. 1 *Block diagram of CSPS method*

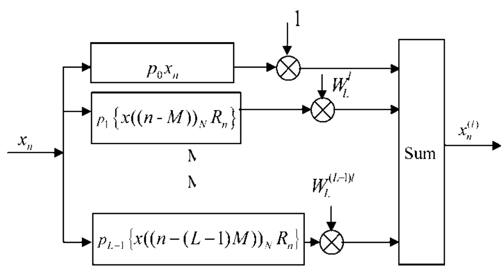

Fig. 2 *Computation of one candidate $x_n^{(l)}$ using (9)*

signal can be recovered in frequency domain by multiplying the received signal with the conjugate of $\boldsymbol{B}_{\text{opt}}$.

The d_i may be chosen with continuous-valued phase angle, but one of more appropriate ways in practical system is a restriction on a finite set of S (e.g. 4) allowed phase angles to reduce the searching complexity. In our simulation, for example, when $S = 4$, the finite sets for each element are $d_0 = 1$, $d_1 \in \{e^{j\pi/6}, e^{j4\pi/6}, e^{j7\pi/6}, e^{j10\pi/6}\}$, $d_2 \in \{1, j, -1, j\}$, $d_3 \in \{e^{j2\pi/6}, e^{j5\pi/6}, e^{j8\pi/6}, e^{j11\pi/6}\}$, respectively. Note that one weight can be fixed without any performance loss.

In this method, the elements in the phase sequences are optimised to create the optimal phase sequence, hence named the optimised CSPS (OCSPS).

4 Comparisons of CSPS, OCSPS, SLM and PTS

In this section, the comparison of four methods, CSPS, OCSPS, PTS and SLM is made in terms of their relationship, the size of their respective SI and the computational costs.

4.1 Relationship of the four methods

When the parameter L in CSPS is equal to M_1 in SLM, that is $L = M_1$, the two methods have same number of phase sequences and the candidates. However, since in CSPS, the L-phase sequences are generated by circularly shifting $\boldsymbol{B}$, the phase sequences are not statistically independent. Therefore one can say that the CSPS is similar to the SLM except that the L-phase sequences in the CSPS are not statistically independent.

To see the relationship between CSPS and PTS, observe that $\boldsymbol{X}^{(l)}$ in (6) can be rewritten as (see Section 9)

$$X_k^{(l)} = X_k \cdot B_k^{(l)} = \sum_{i=0}^{L-1} P_i^{(l)} \sum_{m=0}^{M-1} X_{mL+i}\delta(k - mL - i) \quad (13)$$

where

$$P_i^{(l)} = P((i+l))_L R_i, \quad l = 0, \ldots, L-1$$

are the circular left-shifted sequences of P_i, $\sum_{m=0}^{M-1} X_{mL+i}\delta(k - mL - i)$ $(i = 0, \ldots, L-1)$ are the sub-blocks in PTS obtained from interleaved partition method [7, 8]. Therefore the CSPS is similar to PTS with fixed weights when the data is partitioned into L sub-blocks by using the interleaved partition scheme.

Comparing (2) and (11), the candidates in OCSPS and PTS are weighted sum of circular shifts of x_n and PTSs, respectively, and both of them need the searching procedures to obtain the optimal weights. M_2 IDFTs are required to obtain the PTSs in PTS, while only one IDFT is needed to obtain $\{x((n - iM)_N)R_n\}$, $(i = 0, \ldots, L-1)$. Hence, when $L = M_2$ and exhaustive searching method is used in both OCSPS and PTS, the computational costs for determining weights are equal. However, in OCSPS, only one IDFT is needed, that is, $(L-1)$ N-point IDFTs are saved as compared to PTS.

Table 1: Side information of the four methods

Method	SI	SI size in bits
SLM	Index of $Q_k^{(\mu)}$	$\log_2 M_1$
PTS	b_m $(m = 2, \ldots, M_2)$	$(M_2 - 1)\log_2 S$
CSPS	Index of $\boldsymbol{B}^{(l)}$	$\log_2 L$
OCSPS	$\boldsymbol{d}_{\text{opt},i}$ $(i = 1, \ldots, L-1)$	$(L-1)\log_2 S$

4.2 SI of the four methods

To recover the transmitted signal, SI must be known exactly on the receiver side. The SI will cause redundancy of the system, so it is desirable that the SI should be as little as possible. Table 1 gives the respective SI of the four methods. To reduce the computational complexity, the phase factors b_m in the PTS [5] is also limited to finite set of S (e.g. 4) allowed phase angles, for example, $b_m \in \{\pm j, \pm 1\}$.

4.3 Computation costs of the four methods

In Table 2, main computations of four methods are given and, for simplicity, only the number of complex multiplications at the transmitter to obtain the candidates is used to indicate the computational costs. Here we assume that exhaust searching method in PTS is employed to optimise the phase factors. In CSPS, to obtain $\tilde{\boldsymbol{x}}$, one N-point IFFT, one L-point IFFT and computation of the L candidates $x_n^{(l)}$ according to (9) are required. To acquire $x_n^{(l)}$, N-complex multiplications are needed in (10a), and, for a specific l, $(L-1)N$ complex multiplications are required to achieve the weighted sum in (10b) (for $l = 0$, no multiplication is required). In OCSPS, the computation for $\bar{x}(n)$ includes one N-point IFFT to obtain $x(n)$, and computation to obtain the S^{L-1} candidates.

5 Simulation results and discussions

To test the performance of CSPS and OCSPS, several simulations are carried out based on more than 10^4 independent OFDM symbols and comparison is made with SLM and PTS. The complementary cumulative distribution function (CCDF = Pr(PAPR > PAPR$_0$)) for the OFDM signal is calculated using MATLAB and given to illustrate the performance of the algorithms.

In CSPS simulation, we use two ways to obtain the phase sequences $\boldsymbol{P}$. One way is to set $P_k = e^{j\varphi_k}$ with φ_k uniformly distributed on $[0, 2\pi)$, and another way is to select $P_k = \exp\{j2\pi k/(2L)\}$, $(k = 0, 1, \ldots, L-1)$. Simulation results show that the performance of the two ways is similar. But in the second way, $\boldsymbol{P}$ need not be stored both

Table 2: Computational complexity of the four methods

Method	Main computation at the transmitter	Number of complex multiplications
SLM	M_1 N-point IFFTs and the generating M_1 candidates	$C_M = M_1 N\left(\frac{1}{2}\log_2 N + 1\right)$
PTS	M_2 N-point IFFTs and the generating $S^{M_2} - 1$ candidates	$C_M = M_2 N\left(\frac{1}{2}\log_2 N + S^{M_2-1}\right)$
CSPS	One N-point IFFT, one L-point IFFT and generating L candidates	$C_M = \frac{N}{2}\log_2 N + LN + (L-1)^2 N + \frac{L}{2}\log_2 L$
OCSPS	One N-point IFFT and generating S^{L-1} candidates	$C_M = N\left(\frac{1}{2}\log_2 N + S^{L-1}\right)$

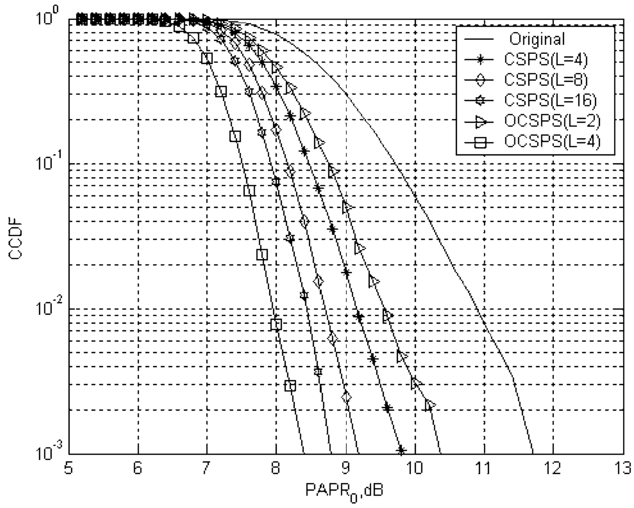

Fig. 3 *CCDF performance of CSPS ($L = 4,8,16$) and OCSPS ($L = 2, 4$) for $N = 256$*

at the transmitter and the receiver only if they know the parameter L. So we use the second way in our simulations. The exhaustive searching is used both in PTS and OCSPS simulation to optimise phase factors b_m $(m = 2, \ldots, M_2)$ and d_i $(i = 1, \ldots, L-1)$, respectively. The sets used for d_i are given in Section 3.3. The sub-blocks used in PTS are obtained by use of the adjacent partitioning method.

5.1 Performance of CSPS and OCSPS

Fig. 3 shows the performance of CSPS and OCSPS with different Ls. The number of sub-carriers is 256. From Fig. 3, it can be seen that for both CSPS and OCSPS, the PAPR can be reduced effectively, the performance increases as L increases and better PAPR reduction performance can be gained in OCSPS. For example, for $L = 4$, comparing with CSPS, OCSPS shows about 1.5 dB gain at 0.1% probability because there are $4^{L-1} = 64$ candidates in OCSPS.

5.2 Performance comparison of CSPS, OCSPS, SLM and PTS

The results of CSPS with $L = 8$ and OCSPS with $L = 4$ for $N = 1024$ are illustrated in Fig. 4, in which the results for

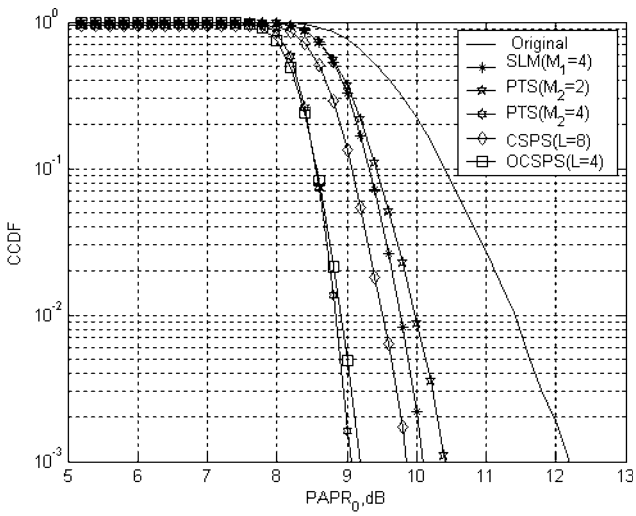

Fig. 4 *CCDF performances of SLM ($M_1 = 4$), PTS ($M_2 = 2$ and 4), CSPS ($L = 8$) and OCSPS ($L = 4$) for $N = 1024$*

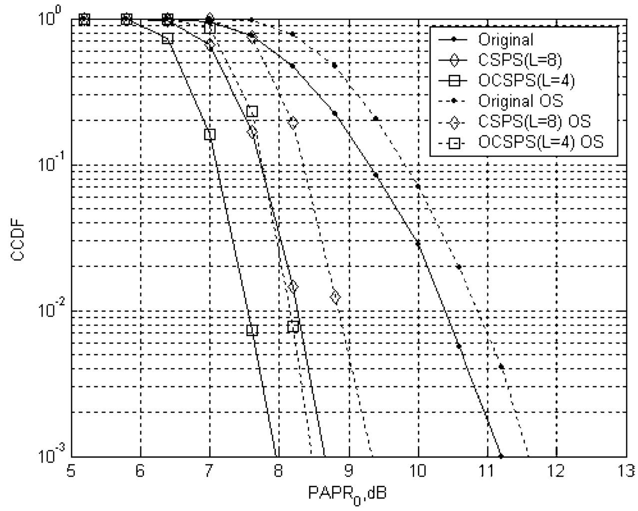

Fig. 5 *Approximate-analogue PAPR performance of CSPS ($L = 8$) and OCSPS ($L = 4$) with oversampling by a factor of 4 for $N = 256$*

SLM with $M_1 = 4$ and PTS with $M_2 = 2$ and 4 are also included for comparison. The results show that the performance of the OCSPS with $L = 4$ is almost the same as that of the PTS with $M_2 = 4$. However, the OCSPS has lower computational costs. Also, the PTS performs better than the SLM when the numbers of IDFT operations are the same in both methods. As comparing the performance of CSPS ($L = 8$) and SLM ($M_1 = 4$), with more candidates, the CSPS method outperforms the SLM, although the eight candidates in CSPS are generated by the use of non-statistically independent phase sequences.

5.3 Approximate-analogue PAPR of the CSPS and OCSPS

It is the large PAPR of an analogue signal that causes problems for amplifiers. To examine the approximate-analogue PAPR reduction using the proposed methods, the oversampled versions of the digital signal can be used. A QPSK-based OFDM signal ($N = 256$) is oversampled by a factor of 4 [9]. For comparison, the performance of the proposed methods for the digital and approximate-analogue signals is shown in Fig. 5. The legends in Fig. 5, for example, 'CSPS ($L = 8$) OS' and 'CSPS ($L = 8$)', indicate the performance of CSPS for $L = 8$ with and without oversampling, respectively. There is a difference of about 0.7 dB between the analogue and digital PAPR for $N = 256$ systems. The PAPR of the approximate-analogue signal is reduced for both the CSPS and OCSPS methods as compared with the original OFDM signal.

6 Conclusion

In this paper, two distortionless methods, CSPS and OCSPS, for PAPR reduction in OFDM are developed, in which a set of CSPS are first designed and used for creating the signal candidates. In the CSPS method, the signal candidates are generated using the TDCC of OFDM data with the IDFT of CSPS, and then the one with the smallest PAPR is selected for transmission. In the OCSPS method, the elements of the final phase sequence used for forming the transmitted signal are optimally selected. The performance of the CSPS and OCSPS methods are evaluated based on simulated data and the comparison with SLM and PTS is conducted in terms of their relationship, SI size and computational costs. Simulation results have shown that

- The CSPS and OCSPS methods both are effective in reducing PAPR of OFDM signals,
- The OCSPS performs even better than the CSPS, and
- The OCSPS, although computational costs are lower, performs as well as PTS.

A distinct feature of the proposed methods is that only one IDFT is needed, and thus, the candidates can be calculated in time domain directly.

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8 References

1 Sternad, M., Ottosson, T., Ahlén, A., and Svensson, A.: 'Attaining both coverage and high spectral efficiency with adaptive OFDM downlinks'. IEEE Vehicular Technology Conf., VTC2003-Fall, Orlando, FL, USA, October 2003

2 Keller, T., and Hanzo, L.: 'Adaptive multicarrier modulation: A convenient framework for time-frequency processing in wireless communications', *Proc. IEEE*, 2000, **88**, (5), pp. 611–640

3 Hu Yu, X., Chen, G., Chen, M., and Gao, X.: 'Toward beyond 3G: the fuTURE project in China', *IEEE Commun. Mag.*, 2005, **43**, (1), pp. 70–75

4 http://www.signal.uu.se/Research/PCCwirelessIP.html

5 Muller, S.H., and Huber, J.B.: 'OFDM with reduced peak to average power ratio by optimum combination of partial transmit sequences', *Electron. Lett.*, 1997, **33**, (5), pp. 368–369

6 Baml, R.W., Fischer, R.F.H., and Huber, J.B.: 'Reducing the peak-to-average power ratio of multicarrier modulation by selected mapping', *Electron. Lett.*, 1996, **32**, (22), pp. 2056–2057

7 Kang, S.G., Kim, J.G., and Joo, E.K.: 'A novel subblock partition scheme for partial transmit sequence OFDM', *IEEE Trans. Broadcasting*, 1999, **45**, (3), pp. 333–338

8 Lu, G., Wu, P., and Carlemalm-Logothetis, C.: 'Enhanced interleaved partitioning PTS for peak-to-average power ratio reduction in OFDM systems', *Electron. Lett.*, 2006, **42**, (17), pp. 983–984

9 Cimini, L.J., and Sollenberger, N.R.: 'Peak-to-average power ratio reduction of an OFDM signal using partial transmit sequences', *IEEE Commun. Lett.*, 2000, **4**, (3), pp. 86–88

10 Hee Han, S., and Hong Lee, J.: 'PAPR reduction of OFDM signal using a reduced complexity PTS technique', *IEEE Commun. Lett.*, 2004, **11**, (11), pp. 887–890

11 Hill, G.R., Faulkner, M., and Singh, J.: 'Reducing the peak-to-average power ratio in OFDM by cyclically shifting partial transmit sequences', *Electron. Lett.*, 2000, **36**, (6), pp. 560–561

12 Lu, G., Wu, P., and Carlemalm-Logothetis, C.: 'Peak-to-average power ratio reduction in OFDM based on the transformation of partial transmit sequences', *Electron. Lett.*, 2006, **42**, (2), pp. 105–106

13 Oppenheim, A.V., and Schafer, R.W.: 'Discrete-time signal processing' (Prentice-Hall, Englewood Cliffs, NJ)

9 Appendix

Derivation of (13):

Rewrite $X_k^{(l)}$ in (6) as

$$X_k^{(l)} = \sum_{i=0}^{N-1} B_k^{(l)} X_i \delta(k-i),\ l = 0, \ldots, L-1 \tag{14}$$

(14) can be expanded as

$$\begin{aligned} X_k^{(l)} = {} & B_0^{(l)} X_0 \delta(k) + B_1^{(l)} X_1 \delta(k-1) + \cdots \\ & + B_{L-1}^{(l)} X_{L-1} \delta(k-L+1) \\ & + B_L^{(l)} X_L \delta(k-L) + B_{L+1}^{(l)} X_{L+1} \delta(k-L-1) + \cdots \\ & + B_{2L-1}^{(l)} X_{2L-1} \delta(k-2L+1) + \cdots \\ & + B_{(M-1)L}^{(l)} X_{(M-1)L} \delta(k-(M-1)L) + \cdots \\ & + B_{ML-1}^{(l)} X_{ML-1} \delta(k-ML+1) \end{aligned} \tag{15}$$

Thanks to the cyclically shifting structure of $B_k^{(l)}$ (see (5)), one may have

$$\begin{aligned} B_i^{(l)} = B_{i+L}^{(l)} = B_{i+2L}^{(l)} = \cdots = B_{i+(M-1)L}^{(l)} = P_i^{(l)},\ i \\ = 0, \ldots, L-1 \end{aligned} \tag{16}$$

where$P_k^{(l)} = P((k+l))_L R_k$, $l = 0, \cdots, L-1$ are the circular left-shifted sequences of $\boldsymbol{P}$. Then, substituting (16) into (15), one can obtain

$$\begin{aligned} X^{(l)} & = P_0^{(l)} \sum_{m=0}^{M-1} X_{mL} \delta(k-mL) + \cdots + P_{L-1}^{(l)} \\ & \quad \times \sum_{m=0}^{M-1} X_{mL+L-1} \delta(k-(m+1)L+1) \\ & = \sum_{i=0}^{L-1} P_i^{(l)} \left(\sum_{m=0}^{M-1} X_{mL+i} \delta(k-mL-i) \right) \end{aligned} \tag{17}$$

where$\sum_{m=0}^{M-1} X_{mL+i} \delta(k-mL-i)$ $(i = 0, \ldots, L-1)$ are the sub-blocks in PTS when interleaved partition method [7, 8] is used.