Peak-to-average power ratio reduction in OFDM based on transformation of partial transmit sequences

G. Lu, P. Wu and C. Carlemalm-Logothetis

A novel scheme (transformation of partial transmit sequences, T-PTS) is proposed for the partial transmit sequence (PTS) method to reduce peak-to-average power ratio of OFDM signals. In T-PTS certain transformations are applied on partial transmit sequences to generate more candidates for increasing performance. Simulations show that T-PTS outperforms conventional PTS for the same number of subblocks; and both schemes have similar performance for the same candidate number, but T-PTS is less complex.

Introduction: To avoid the occurrence of large peak power of signals in orthogonal frequency division multiplexing (OFDM) so as to alleviate the requirement of expensive high linear power amplifiers, various methods for the reduction of peak-to-average-power ratio (PAPR) have been developed during the last decade [1–4]. One of them is the partial transmit sequence (PTS) method [2, 3]. It is known that PTS can improve its performance by increasing the number of candidates. Although dividing the transmitted data into more subblocks may increase the number of candidates, this will result in more IDFT operations and thus increase computational cost. Other methods for increasing the number of candidates were previously proposed, for example, cyclic shifting of the partial transmit sequences (PTS/CSS) [5]. In this Letter, a novel scheme for PTS is proposed in which the number of candidates is increased by making different predefined transformations of the partial transmit sequences (abbreviated as PTSs to denote sequences) and then forming new sets of PTSs from the original and transformed ones. The performance is, thus, improved.

Background—conventional PTS: A frequency domain data of an OFDM system with N subcarriers is denoted X_k. X_k is partitioned into V disjoint subblocks, $X_k^{(v)}$ ($v=1,\ldots,V$; $k=0,\ldots,N-1$), in which all the subcarriers that are occupied by the other subblocks are set to zero, so that $X_k=\sum_{v=1}^{V}X_k^{(v)}$. By applying a phase-rotation factor $b_v\in\{\pm j,\pm 1\}$ to the vth subblock $X_k^{(v)}$ one may create alternative frequency domain signals as

$$\widehat{X}_k=\sum_{v=1}^{V}b_vX_k^{(v)} \tag{1}$$

The IDFTs of $\widehat{X}_k$ give the candidates $\widehat{X}_n=IDFT\{\widehat{X}_k\}=\sum_{v=1}^{V}b_vx_n^{(v)}$, where $x_n^{(v)}=IDFT\{X_n^{(v)}\}$ are the V so-called PTSs. Note that the aggregate number of candidates in this case is $4^{(V-1)}$. The transmit sequence $\hat{x}_n=\sum_{v=1}^{V}\hat{b}_vx_n^{(v)}$ (with the smallest PAPR) selected for transmission is determined based on

$$\{\hat{b}_1,\hat{b}_2,\ldots,\hat{b}_V\}=\underset{\{b_1,b_2,\ldots,b_V\}}{\arg\min}\left\{\max_{0\le n\le N-1}|\hat{x}_n|\right\} \tag{2}$$

Transformation of partial transmit sequences (T-PTS): To increase the number of candidates for a given number of subblocks $X_k^{(v)}$ ($v=1,\ldots,V$), alternative frequency domain signals different from those in (1) can be generated in the following way:

$$\tilde{X}_k=\sum_{v=1}^{V}b_vT^{(v)}\left[X_k^{(v)}\right] \tag{3}$$

where $T^{(v)}[X_k^{(v)}]$ is a certain predefined transformation (but *not* IDFT!) made on $X_k^{(v)}$. Then the candidates (in the time domain) can be written as

$$\tilde{x}_n=\sum_{v=1}^{V}b_vIDFT\left\{T^{(v)}\left[\tilde{X}_k^{(v)}\right]\right\}=\sum_{v=1}^{V}b_vx_{n,T}^{(v)} \tag{4}$$

where $x_{n,T}^{(v)}=IDFT\{T^{(v)}[X_k^{(v)}]\}$ are the IDFTs of the transformed subblocks.

Without loss of generality, one may choose to perform transformations only on some of the subblocks. For example, no transformation is made on the first subblock, and the weighting factor is chosen to be $b_1=1$. Then the candidates in (4) can written as

$$\tilde{x}_n=x_n^{(1)}+\sum_{v=2}^{V}b_vx_{n,T}^{(v)} \tag{5}$$

In principle, the transformation can be made in many different ways to get different sets of transformed PTSs. In practice, one uses simple trivial transformations like conjugate operation, frequency reversal operation, circular shift operation, and their combinations. For such transformations, one can make use of DFT properties (the transform pairs) to find $x_{n,T}^{(v)}$ directly from $x_n^{(v)}$ in the time domain instead of from $IDFT\{T^{(v)}[X_k^{(v)}]\}$ so as to avoid additional IDFTs.

Four such simple, elementary transformations are defined as follows:

$T_1[\cdot]$ – the conjugate of the subblock $X_k^{(v)}$:

$$T_1\left[X_k^{(v)}\right]=\left(X_k^{(v)}\right)^*\Leftrightarrow x_{n,T}^{(v)}=\left(x_{N-n}^{(v)}\right)^*$$

$T_2[\cdot]$ – the frequency reversal of $X_k^{(v)}$:

$$T_2\left[X_k^{(v)}\right]=X_{N-k}^{(v)}\Leftrightarrow x_{n,T}^{(v)}=x_{N-n}^{(v)}$$

$T_3[\cdot]$ – the circular shift of $X_k^{(v)}$ by l:

$$T_3\left[X_k^{(v)}\right]=X_{k-l}^{(v)}\Leftrightarrow x_{n,T}^{(v)}=x_n^{(v)}e^{j2\pi nl/N}$$

$T_4[\cdot]$ – the multiplication of $X_k^{(v)}$ with a linear phase sequence:

$$T_4\left[X_k^{(v)}\right]=X_k^{(v)}P_k$$

where $P_k=e^{-j2\pi km/N}$, $k=0,\ldots,N-1$, then $x_{n,T}^{(v)}=x_{n-m}^{(v)}$.

These elementary transformations can be used individually or jointly on different subblocks. For various combinations of two transformations, $T_i[\cdot]$ and $T_j[\cdot]$, the notation $T_{ij}[\cdot]=T_i[T_j[\cdot]]$ ($i,j=1,\ldots,4$) is used, and for combinations of three transformations, $T_{ijk}[\cdot]=T_i[T_j[T_k[\cdot]]]$ is used, and so on. Examples and their corresponding time domain counterparts are

$$T_{41}\left[X_k^{(v)}\right]=\left(X_k^{(v)}\right)^*e^{-j2\pi km/N}\Leftrightarrow x_{n,T}^{(v)}=\left(x_{m-n}^{(v)}\right)^*$$
$$T_{423}\left[X_k^{(v)}\right]=X_{N-k+1}^{(v)}e^{-j2\pi km/N}\Leftrightarrow x_{n,T}^{(v)}=x_{N-n+m}^{(v)}e^{j2\pi(n-m)l/N}$$
$$T_{4123}\left[X_k^{(v)}\right]=\left(X_{N-k+1}^{(v)}\right)^*e^{-j2\pi km/N}\Leftrightarrow x_{n,T}^{(v)}=\left(x_{n-m}^{(v)}\right)^*e^{j2\pi(n-m)l/N}$$

Remark 1: If only using $T_4[\cdot]$, T-PTS becomes PTS/CSS [5], which is thus a special case of T-PTS.

Remark 2: In $T_4[\cdot]$, when m is selected from the four element set $\{0,\ N/4,\ N/2,\ 3N/4\}$, $P_k=e^{-j2\pi km/N}$ becomes 1, $(-j)^k$, $(-1)^k$ and j^k, respectively, which results in four different transformed subblocks $X_k^{(v)}$, $(-j)^kX_k^{(v)}$, $(-1)^kX_k^{(k)}$ and $j^kX_k^{(v)}$.

Remark 3: It should be noted that, for combined transformations $T_{423}[\cdot]$ and $T_{4123}[\cdot]$, care should be taken for selecting l: (i) avoiding no overlap of the transformed subblock with the other subblocks, and (ii) avoiding no additional computations to obtain $x_{n,T}^{(v)}$. For example, for $V=2$, if $\{X_0,\ldots,X_{N/2-1},X_{N/2}\}$ is allocated to the first subblock and the remaining data to the second, choosing $l=N/2$ can meet the requirements.

Following the above example, when $V=2$, one can get 16 alternative representations of the second subblock, $T_4[X_k^{(2)}]$, $T_{41}[X_k^{(2)}]$, $T_{423}[X_k^{(2)}]$, $T_{4123}[X_k^{(2)}]$. Since b_2 is selected from the four element set $\{\pm j,\ \pm 1\}$, up to 64 candidates can be created, similar to conventional PTS with $V=4$.

Since different transformations can be made on different subblocks, the T-PTS scheme is able to create many different candidates when V becomes larger, e.g. 3, 4, etc.

Simulations and results: In simulations the complement cumulative distribution function (CCDF) of the PAPR of OFDM signals with $N=128$, $CCDF=\Pr\{PAPR>PAPR_0\}$, is used to evaluate the performance of T-PTS in comparison with conventional PTS and PTS/CSS schemes.

Fig. 1 shows the CCDFs of the PAPR of QPSK-modulated OFDM signals in PTS, PTS/CSS and T-PTS, respectively. The number of candidates for PTS, PTS/CSS and T-PTS is 4, 16 and 64, respectively, for $V=2$. Clearly, the method with more candidates outperforms the one with fewer candidates. For instance, 0.1% PAPR of the normal OFDM is about 11.4 dB, and those of PTS ($V=2$), PTS/CSS and

T-PTS are improved by about 2, 3.2 and 3.8 dB, respectively. For the same number of candidates (but different subblocks), T-PTS ($V = 2$) has similar performance to PTS ($V = 4$), but T-PTS uses two fewer IDFTs, and thus has lower computational cost than PTS.

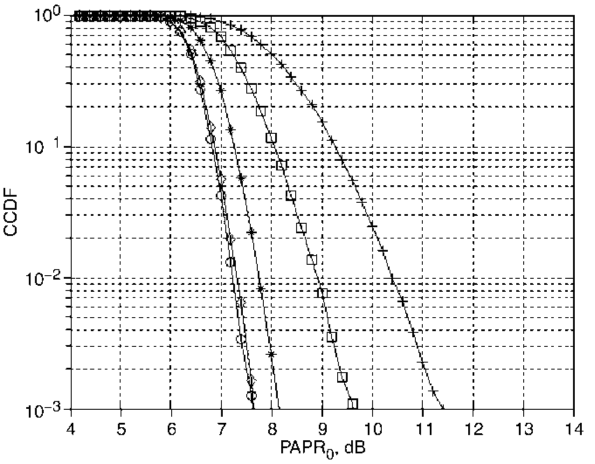

Fig. 1 *CCDFs of PTS, PTS/CSS and T-PTS*

—+— normal OFDM
—□— PTS ($V = 2$)
—∗— PTS/CSS ($V = 2$)
—◇— PTS ($V = 4$)
—○— T-PTS ($V = 2$)

Conclusions: The proposed scheme (T-PTS) can effectively increase the number of candidate signals by transforming PTSs in different ways. When using the same number of subblocks, therefore, T-PTS may have more candidates available than the conventional PTS so that T-PTS performs better. When the same number of candidates is used, T-PTS has similar performance to PTS, but has less computational complexity because of fewer IDFT operations. Conventional PTS, PTS/CSS scheme and other existing PTS schemes may be unified under the T-PTS scheme, which is, thus, a more general one.

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