Analysis of Intermodulation Distortion on Log-Normal Shadowed WLAN Channels

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Abstract

In wireless local area network systems using multicarrier modulation, intermodulation distortion (IMD) occurs due to the nonlinear transmit amplifier and the non-constant envelope of the transmitted signals. In the Letter, a closed form expression for the blocking probability assuming a log-normal distributed fading channel is presented. The result is verified using Monte Carlo simulations.

1 Introduction

Multicarrier modulation techniques as orthogonal frequency division multiplexing (OFDM) are often proposed in wireless local area networks (WLANs) to mitigate the multipath and time dispersive channels of high-bit-rate indoor systems. The multicarrier technique is effective since each sub-carrier is modulated with a low bit rate, thus resulting in a symbol period much longer than typical echo delays. Any multicarrier signal has a large peak-to-mean envelope power ratio and when passed through a nonlinear device, such as a transmitter power amplifier, intermodulation distortion (IMD) is generated. Due to the near-far ratio of two users connected to a WLAN access point (AP), the IMD generated by a near transmitter might block a far transmitter. This blocking will manifests itself as a reduction in system capacity from the ideal case.

Often, in a WLAN environment, there is a shadow fading due to obstacles as office furniture and walls that fades the signals on a large scale. It is often modeled as a log-normal distribution around the mean given by a path loss equation.

The question to be answered in this letter is what level of IMD can be tolerated for a given allowed probability of blocking. The problem was earlier addressed in [1] where a closed form expression was derived that characterize the relationship between the blocking probability and linearity requirements of the power amplifier in a simple channel with an inverse power law attenuation of the signal power. In this letter, we present the corresponding closed form expression for the blocking probability in the log-normal fading case. The presented results are verified by Monte Carlo simulations.

2 Analysis

Our assumptions are similar to the assumptions in [1] where an AP is the receiver (RX) and the desired transmitter TX_d and an interfering transmitter TX_i are situated on the distance r_d and r_i from the AP respectively, see Figure 1. The maximum range for the AP coverage is D. In the analysis of this problem we make some assumptions to make the problem analytically tractable. We assume that the transmitters are transmitting with the same power and without power control. The transmitter positions are independent and uniform area distributed over a disc of radius D from the AP. When the desired signal power to IMD power ratio is below a certain threshold α, we assume that the desired transmitter is completely blocked.

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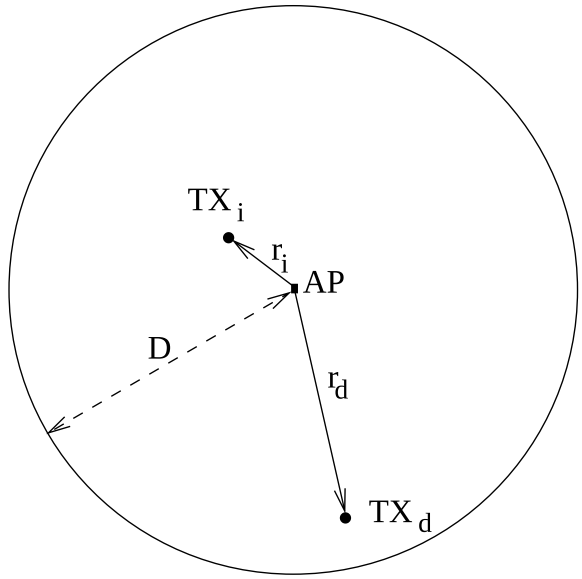

Figure 1: The near far scenario where IMD from TX_i is interfering the signal from TX_d at the AP. D is the maximum range of the AP.

If we assume a simple power law equation with decay index γ, the expectation of the received desired and IMD power at the AP is

$$\begin{aligned} m_d &= E\{p_d\} = 1/r_d^{\gamma} \\ m_i &= E\{p_i\} = \beta/r_i^{\gamma} \end{aligned} \tag{1}$$

where $-10\log_{10}(\beta)$ dBc is the IMD product level transmitted from TX_i and we have normalized the transmitted power from TX_d to 1 without loss of generality.
At the AP, we define the blocking probability as

$$P(blocking) \triangleq Pr\{p_d/p_i < \alpha\} \tag{2}$$

where p_n and p_i are the received desired and IMD power respectively averaged over the eventual fast fading. The specified threshold α depends on the tolerance of the modulation to interference. The received power depends on the distances r_d and r_i, the path loss and on the probability distribution of the fading. Using Bayes theorem we rewrite the blocking probability (2) as

$$Pr\{p_d/p_i < \alpha\} = (2\pi)^2 \int_0^D \int_0^D Pr\{p_d/p_i < \alpha | r_d, r_i\} p(r_d)p(r_i)dr_d dr_i \tag{3}$$

where the factor $(2\pi)^2$ is due to the integrated uniform angle distribution and we assumed independence of the TX_d and TX_i positions. The marginal distribution of the radial position in an area uniform PDF is

$$p(r) = \frac{2r}{D^2}. \tag{4}$$

The blocking probability conditioned on the area mean power m_d, m_i for a log-normal shadowed channel with equal standard deviations for the desired and the IMD signal fading statistics, $\sigma_d = \sigma_i = \sigma$, can be written as [2]:

$$Pr\{p_d/p_i < \alpha | m_d, m_i\} = Q\left(\frac{\ln\left(\frac{m_d}{\alpha m_i}\right)}{\sqrt{2}\sigma}\right) \tag{5}$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2} dt. \tag{6}$$

The blocking probability is now obtained using (1),(4),(5) in (3) as

$$Pr\left\{p_d/p_i < \alpha\right\} = \frac{16\pi^2}{D^4} \int_0^D \int_0^D Q\left(\frac{\ln\left(\left(\frac{r_i}{r_d}\right)^{\gamma}/\beta\alpha\right)}{\sqrt{2}\sigma}\right) r_i r_d dr_i dr_d \tag{7}$$

Equation (7) reduces to

$$Pr\left\{p_d/p_i < \alpha\right\} = Q(c) + \frac{1}{2} e^{\frac{2\sigma^2}{\gamma^2}} \left[(\beta\alpha)^{2/\gamma} Q\left(\frac{2\sigma}{\gamma} - c\right) - \frac{1}{(\beta\alpha)^{2/\gamma}} Q\left(\frac{2\sigma}{\gamma} + c\right)\right] \tag{8}$$

where $c = -\ln(\beta\alpha)/\sigma$. Note that this expression is independent of the maximum range D. In the limit $\sigma \to 0$, i.e. the non-fading case, the expression (8) approaches the blocking probability in [1] for the simple power law decay of the signals, if we assume $\beta < 1/\alpha$ which is true for practical IMD levels:

$$\lim_{\sigma \to 0} Pr\left\{p_d/p_i < \alpha\right\} = \frac{1}{2}(\beta\alpha)^{2/\gamma} \tag{9}$$

If we assume that the desired to interference level tolerance of the modulation is α=15 dBc and plot the blocking probability as a function of the IMD level β, we get the family of curves in Figure 2 for different values of the log-normal PDF parameter σ. The path loss index was set to $\gamma = 4$. To verify the expression (8), a Monte Carlo simulation was performed by randomly placing terminals around the AP and selecting a shadowing component from a log-normal distribution. Then equation (2) is evaluated to determine whether blocking has occurred. We see that the correspondence between the theory and simulated results are satisfactory.

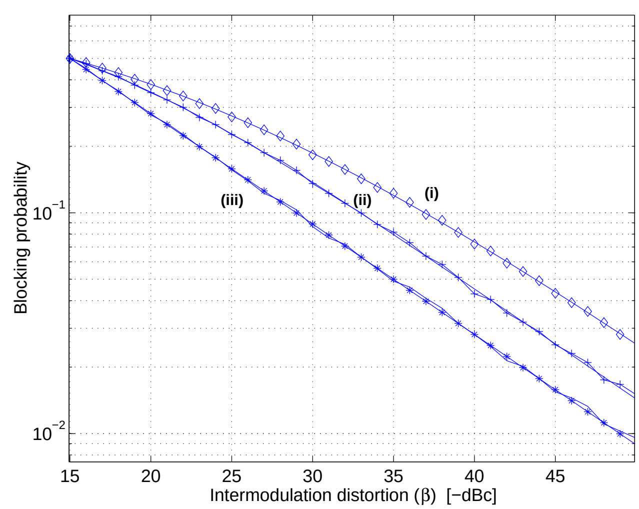

Figure 2: Theoretical (solid) and results from Monte Carlo simulation (marked) of channel blocking probability with respect to linearity of the transmit power amplifier. Different cases of log-normal fading (i) σ=9 dB, (ii) σ=6 dB, (iii) $\sigma=-\infty$ dB. (no fading)

In Figure 2 the curve for the non-fading case is displayed, and it shows the lowest probability of blocking for a given IMD level. Hence, the shadow fading increases the blocking probability. When the IMD level equals the IMD tolerance ($\beta\alpha$=1)the blocking probability is 0.5, because blocking will the occur when TX_i is closer to the AP than TX_d and this occurs at equal probabilities $Pr(r_d < r_i) = Pr(r_i < r_d) = 0.5$.
It can be seen from Figure 2 that for a blocking probability of 10%, β is -37 dBc, -33 dBc with a shadow fading standard deviation of σ=9 dB and 6 dB respectively.

3 Conclusion

An exact formula for the blocking probability as a function of IMD level in a multicarrier radio LAN system subject to log-normal distributed shadow fading was presented. The formula was verified using Monte Carlo simulations.

References

[1] M.Li, T.A.Wilkinson, M.Beach, S.K.Barton, H.Xue, I.R.Johnson, and A.Nix, "Analysis of intermodulation distortion specification for radio LANs using multicarrier schemes," *Electronic Letters*, vol. 29, pp. 1229–1231, 1993.

[2] A.Kegel R.Prasad, "Improved assessment of interference limits in cellular radio performance," *IEEE Transactions on Vehicular Technology*, vol. 40, pp. 412–419, 1991.