Analysis of Quantisation Effects in Adaptive Array Antennas

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Abstract

Adaptive antennas are used to increase the spectral efficiency in mobile telecommunication systems. The precision required in the beamforming and in the calibration process is evaluated by simulations and verified by measurements on an adaptive antenna testbed. As a performance measure, the carrier to interference ratio (CIR) on adaptive antenna output is used. A theoretical expression for the output CIR is derived and compared with simulations and measurements. The testbed is designed for the DCS-1800 system and operates in the uplink only. It uses the hybrid-digital-analog beamformer, where the weights are computed in a DSP, but the beamforming is performed on RF or IF frequency using analog phase-shifters and attenuators. We show how the simulation tool can be used to make the design balanced and thus avoiding over- or under-dimensioning of hardware parameters such as weight accuracy and calibration accuracy. This is important for the system-designer, and can be used to find performance bottlenecks.

1 Introduction

The maximum achievable CIR of an adaptive array antenna is affected by many parameters as the power of the received signals, their angle of arrival, the number of antennas and their position [1],[2]. The topic of this paper is to investigate how the implementation of the signal processing in hardware affects the performance of the adaptive array antenna.

The paper considers a hybrid-digital-analog beamformer (ABF), where the adaptive algorithm is implemented in a digital signal processor (DSP), but the beamformer weights are implemented in hardware. Thus, the beamforming is carried out on the RF or IF frequency, using digitally controlled analog phase-shifters and attenuators. The benefits with the ABF is that it can be used as an add-on system to existing basestations, to boost capacity in "hot-spot" traffic areas.

To experimentally investigate the performance of an adaptive antenna array in a mobile communication system, an adaptive antenna testbed has been designed and constructed. It is designed for use in the DCS-1800 system. The testbed operates in receiving mode only and is of the described ABF type.

The testbed does not allow us to manipulate the weight accuracy or the number of bits used in the analog to digital converters (ADCs). Thus, to study the effect of implementation, and to compare with the measured results from the testbed, a simulation model was developed. The simulation model of the ABF consider the limitations of the following adaptive antenna parts:

Sampling receivers The sampling receivers down-convert the signal to a complex baseband signal, and ADC sample the signal. If the received signal amplitude exceeds the maximum amplitude level of the ADC input, clipping will occur, and performance will quickly degrade. The ADCs will also introduce quantisation noise with a variance that should be in the same order of magnitude or less as the thermal noise variance for best utilization of the dynamic range. So by using more bits in the ADCs the dynamic range is increased and signals with larger amplitudes can be received without saturating the ADCs.

DSP The DSP has a finite word length that affects the the numerical stability and accuracy of matrix inversions used by some algorithms. The required word-length to achieve desirable performance by using the optimal weights has been studied by Nitzberg [3], who showed that the single interference source case requires the highest precision in the DSP (the largest DSP word-length). Also, algorithm-specific errors is introduced that affect performance, as the number of samples used in the estimation of the covariance matrix. This has been studied by Monzingo and Miller [2].

Weighting units The weighting units attenuates and phase shifts the received RF signals to create the beam-pattern. The weighting units have finite accuracy determined by the type of weight used, and the number of control bits from the DSP.

Calibration To match the different antenna channels, a calibration must be performed. Depending on the calibration method used, there will be an amount of inaccuracy in the calibration process, due to e.g. quantisation. Also, time-varying calibration errors will occur when the antenna is subject to e.g. temperature and humidity variations or mechanical vibrations. In [4] an on-line calibration algorithm is presented that is transparent to main antenna operation, continuously tracking the changes in antenna channels.

The novel aspect of this paper is to investigate these errors in the framework of mobile communication systems, and comparisons can be made with the adaptive antenna testbed to verify the simulations. Also the dynamic range of the sampling receiver is included, which has not been considered in previous papers. A theoretical model is developed, and the theoretical results are compared to the simulations and testbed results.

2 Preliminary consideration

A signal model of the hardware is described in this Section. Figure 1 shows the equivalent signal model used. We assume that all signals are represented by their complex baseband equivalent.

The noise of the amplifiers in the front-end and the spatially and temporally white noise received by the N antennas are modeled as an equivalent noise source $\mathbf{n}_t(t)$. We assume that $\mathbf{n}_t(t)$ is a $N \times 1$ vector of zero mean, complex Gaussian distributed variables, and with covariance matrix $\sigma_t^2\mathbf{I}$.

We assume that two narrowband signals impinges on an uniform linear array (ULA) antenna from two distinct directions θ_d and θ_i. The received signal by the N antennas can then be described by the $N \times 1$ vector $\mathbf{x}(t)$ as

$$\mathbf{x}(t) = \mathbf{a}(\theta_d)s_d(t) + \mathbf{a}(\theta_i)s_i(t) + \mathbf{n}_t(t) \quad (1)$$

where $\mathbf{a}(\theta_d)$ is the array response vector in direction θ_d including antenna element gain and polarization. The signals $s_d(t)$ and $s_i(t)$ are the desired and interferer signals respectively.

The receivers will introduce noise in the down-converting process and the limited dynamic range of the ADC will set the maximal distortion-free amplitude before clipping of the received signal occurs. The ADC also introduces quantisation noise. The total noise level will set the minimum detectable signal amplitude of the receiver. In the signal model, the sampling receiver (SRX) are replaced by limiters and a noise source $\mathbf{n}_r(t)$. This noise represent the internal noise generated in the receivers and the quantisation noise generated in the sampling process. The corresponding signal vector in the DSP can thus be written as

$$\mathbf{x}_{DSP}(t_k) = \mathrm{sat}\left[\mathbf{a}(\theta_d)s_d(t_k) + \mathbf{a}(\theta_i)s_i(t_k) + \mathbf{n}_r(t_k) + \mathbf{n}_t(t_k)\right] \quad (2)$$

where the index k have been inserted to the express the sampling timings at $t = t_k$. The saturation operator, $sat[\cdot]$, limits the signals, where the limiting amplitude level is dependent on the automatic gain control (AGC) in the receivers and the dynamic range of the ADC.

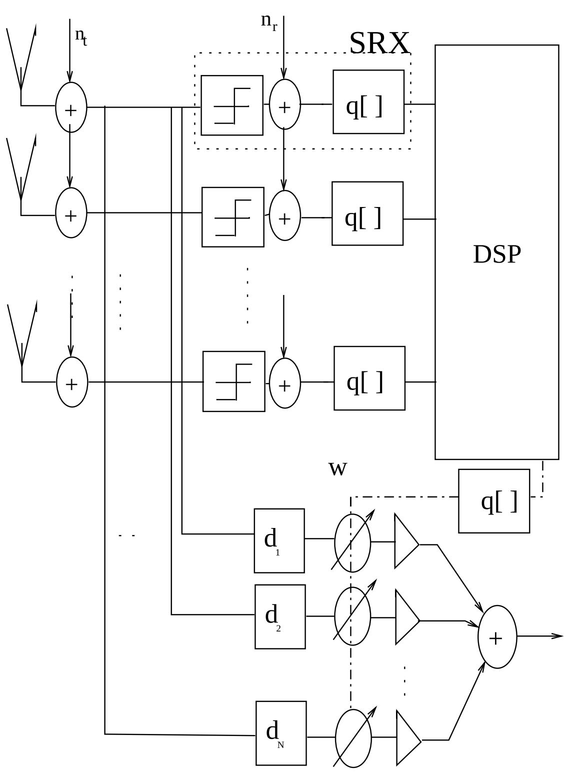

Figure 1: Signal model of receiving array antenna

2.1 Calibration

The aim of the calibration is to measure the transfer function of the channel between the SRX and the weighting units. This is important because the weights are calculated based on the signals at the SRX but applied to the signals at the weighting units. The signals at the weighting units are assumed to be an magnitude and phase-shifted replica of the signals at the SRX. The transfer functions are used to compensate the weights before they are applied in the weighting of the signals. By introducing the complex diagonal matrix $\mathbf{C} = \mathrm{diag}(c_1, \ldots, c_N)$, the relation of the signal at the weighting units, $\mathbf{x}_w(t)$, and the SRX, $\mathbf{x}(t)$, can be expressed as $\mathbf{x}_w(t) = \mathbf{C}\mathbf{x}(t)$.

Assume that the algorithm calculates the weight vector $\mathbf{w}_{DSP}$, based on the received signals $\mathbf{x}_{DSP}(t_k)$. To compensate for the differences in the receiving channels, the weights are pre-adjusted to

$$\mathbf{w} = (\mathbf{C}^{-1})^H \mathbf{w}_{DSP} \quad (3)$$

where $(\cdot)^H$ denotes hermitian transpose. The analog beamformer output signal $y(t)$ will be

$$y(t) = \mathbf{w}^H \mathbf{x}_w(t) = \mathbf{w}_{DSP}^H \mathbf{C}^{-1}\mathbf{C}\mathbf{x}(t) = \mathbf{w}_{DSP}^H \mathbf{x}(t) \quad (4)$$

Thus the effect of the transfer function $\mathbf{C}$ is canceled. We assume that $\mathbf{C}$ is constant over the receiver passband of interest.

The calibration have a finite accuracy, so $\mathbf{C}$ is not known exactly. Also, due to temperature drift and aging, the calibration correction matrix will not describe the actual transfer function. Let the matrix $\tilde{\mathbf{C}}$ be the calibration correction matrix that is stored in the DSP. If the calibration is error-free, $\tilde{\mathbf{C}} = \mathbf{C}$. We can now write $\tilde{\mathbf{C}}^{-1}\mathbf{C} = \mathbf{I} + \delta\mathbf{C}$, where $\delta\mathbf{C}$ is a diagonal matrix with complex elements, representing the relative errors in the calibration. Writing the elements of $\delta\mathbf{C}$ as $\delta c_l = d_l e^{j\phi_l}$, we separate the relative calibration errors into a magnitude error d_l and a phase error ϕ_l. The magnitude d_l is assumed to be bounded in the range $[\pm\varepsilon_a]$ and the phase error ϕ_l is in the range $[\pm\varepsilon_\phi]$. Thus an upper bound on the squared error of the relative calibration error can be shown to be $c_{max}^2 = \varepsilon_a^2 + \varepsilon_\phi^2$.

2.2 The beamformer output signal

The DSP calculates the weights by using the sample matrix inverse (SMI) algorithm [2], using the 26 bit training sequence in the mid-amble of a DCS-1800 timeslot. Due to the finite step size in the hardware weighting units, the weights $\mathbf{w}$ will be quantised and an error vector δ_w is introduced. The phase and magnitude of the weight quantisation errors are assumed to be independent and zero mean, white, and uniformly distributed variables in the ranges $[\pm\varepsilon_M]$ and $[\pm\varepsilon_\theta]$ respectively. The total weight error covariance matrix is $\sigma_w^2\mathbf{I}$, where σ_w^2 can be shown to be upper bounded by $\sigma_M^2 + w_{max}^2\sigma_\theta^2$, where $\sigma_M^2 = \varepsilon_M^2/3$ and $\sigma_\theta^2 = \varepsilon_\theta^2/3$ and w_{max} is the maximal weight magnitude [5]. The signal on the beamformer output can be written as

$$y(t) = (\mathbf{w} + \delta_w)^H (\mathbf{I} + \delta\mathbf{C})\mathbf{x}(t) \quad (5)$$

Using equation (5) and assuming that the stochastic vectors δ_w and $\mathbf{x}(t)$ are independent, the power of the beamformer output signal can be written as

$$\begin{aligned} E\left\{|y(t)|^2\right\} = {} & \mathbf{w}^H E\left\{\mathbf{x}(t)\mathbf{x}^H(t)\right\}\mathbf{w} + \\ & + E\left\{\delta_w^H \mathbf{x}(t)\mathbf{x}^H(t)\delta_w\right\} + \\ & + \mathbf{w}^H E\left\{\delta\mathbf{C}\mathbf{x}(t)\mathbf{x}^H(t)\delta\mathbf{C}^H\right\}\mathbf{w} + \\ & + E\left\{\delta_w^H \delta\mathbf{C}\mathbf{x}(t)\mathbf{x}^H(t)\delta\mathbf{C}^H\delta_w\right\} \end{aligned} \quad (6)$$

Where the terms in (6) is referred to as the first, second, third and fourth term. The first term is the output signal from a beamformer without calibration or weight quantisation errors. The second term is due to weight quantisation errors. The third term in (6) includes the diagonal matrix $\delta\mathbf{C}$ that models the calibration errors and the fourth term contains the product of the two error variables and is neglected in the following. We assume that the first term includes the desired, interferer and noise signals, and the other terms includes the noise and interfering signals in the total output signal. This is a pessimistic assumption, because these terms also includes some signal which is correlated with the desired signal.

The output carrier to interferer plus noise ratio (CINR) can now be derived by assuming that the finite sample size effect and the saturation of the SRX discussed above is not taken into consideration. By carrying out the expectations in (6), a lower bound on the achievable output CINR can be expressed as

$$\begin{aligned} & \text{CINR} \geq \mathbf{w}^H\mathbf{R}_d\mathbf{w} / \left[\mathbf{w}^H\mathbf{R}_i\mathbf{w} + N\sigma_t^2 w_{max}^2 + \right. \\ & \left. + (\sigma_w^2 + Nc_{max}^2 w_{max}^2)(\sigma_d^2|\mathbf{a}(\theta_d)| + \sigma_i^2|\mathbf{a}(\theta_i)| + N\sigma_t^2)\right] \end{aligned} \quad (7)$$

where $\mathbf{R}_i, \mathbf{R}_d$ is the autocorrelation matrices for the weighted desired and interfering signal respectively. This expression illustrates how the desired and interfering signal power (σ_d^2, σ_i^2) "leaks" through the beamformer, due to calibration errors and weight quantisation. In the limit where calibration errors and weight quantisation errors approaches zero, the CINR approaches the well known expression for CINR of a digital beamformer, CINR$\simeq \mathbf{w}^H\mathbf{R}_d\mathbf{w}/\mathbf{w}^H(\mathbf{R}_i + \mathbf{I}\sigma_t^2)\mathbf{w}$.

3 The adaptive antenna testbed

In this section the adaptive antenna testbed is described briefly. For a detailed description, see [6]. The weighting units has a step size of 1° and uses logarithmic attenuators with 1 dB step size and a 50 dB dynamic range. Calibration of the antenna array is performed prior to normal operation of the testbed and the calibration have an accuracy of 1° in phase and 0.75 dB in magnitude.

Measurements were performed in a lab, to validate the performance of the adaptive array antenna in an easy controllable signal environment. The front end array antenna is replaced by an 8×8 Butler matrix. Two signal generators were connected to the Butler matrix to emulate signals impinging ideally from $-61°$ and $-7.2°$.

In practice, the CINR in equation (7) cannot be measured, due to the inability to separate the desired signal from the interfering and noise signal. Instead, we measured a modified carrier to interference ratio on the adaptive antenna output, denoted CIR$_{out}$, in fact, we measured the carrier plus noise to interferer plus noise ratio, where "noise" includes the thermal noise, calibration error noise and the weight quantisation noise.

If the angle of arrival separation between the two signal sources is large enough the expression (7) can be used to approximate the measured quantity CIR$_{out}$ as

$$\begin{aligned} \text{CIR}_{out} \simeq {} & \left[K\sigma_d^2 + N\sigma_t^2 w_{max}^2 + \right. \\ & \left. (\sigma_w^2 + Nc_{max}^2 w_{max}^2)(\sigma_d^2|\mathbf{a}(\theta_d)| + N\sigma_t^2)\right] \\ & / \left[N\sigma_t^2 w_{max}^2 + \right. \\ & \left. (\sigma_w^2 + Nc_{max}^2 w_{max}^2)(\sigma_i^2|\mathbf{a}(\theta_i)| + N\sigma_t^2)\right] \end{aligned} \quad (8)$$

where K is the array gain, including amplifiers in the front end and in the weighting units.

4 Simulation and measurement results

The simulation was performed to imitate the testbed as much as possible. To verify the theoretical model and the simulation results, a comparison is made in Figure 2. The simulation parameters are set equal to the parameters used in the adaptive antenna testbed, e.g. 8 bit ADCs and 1° and 1 dB weight accuracy.

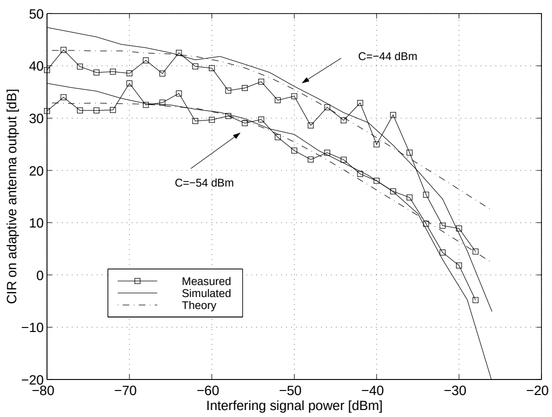

Figure 2: CIR_{out} as a function of interferer power. Comparisons between measured, simulated and theory (equation (8)). Carrier power constant at -44 dBm and -54 dBm. Noise level at -76 dBm.

Figure 2 shows the measured output CIR, denoted CIR_{out} as a function of interfering signal power, when the desired signal power was held constant at two different levels, -44 dBm and -54 dBm. The figure also shows the corresponding simulation results and the theory, described by equation (8). The measured curves are a mean values over ten measurements and the simulated are mean over 100 simulations. The standard deviation is 7 dB and 0.8 dB in measurements and simulations respectively. The theoretical expression does not consider the limited dynamic range of the ADC, so the theoretical CIR_{out} is larger than the measured and simulated CIR_{out} when the interfering signal saturates the ADC.

The theory, equation (8), predicts the CIR_{out} to reach a constant level when the interferer is decreased below the noise level. This is verified by the measurements and the level is determined by the desired signal power and the weight and quantisation error variances.

4.1 Calibration errors and weight accuracy

We investigate how the calibration accuracy affects CIR_{out} in Figure 3. This figure displays CIR_{out} as a function of weight error variance, for four different calibration accuracies. A weight error variance less than 10^{-5} will not further improve the CIR because the calibration errors limits the maximum achievable CIR. To compare with the testbed, with weight variance $1.25 \cdot 10^{-3}$ the maximum achievable CIR_{out} in Figure 3 is approximately 24 dB if the calibration is done without errors. This should be compared to the measured 18 dB from the testbed in the same conditions. Thus, to improve the testbed performance, effort should be put to improve the calibration algorithm, in favor for improving weight accuracy. Improving weight accuracy will raise CIR_{out} only a few dB. The effect of calibra-

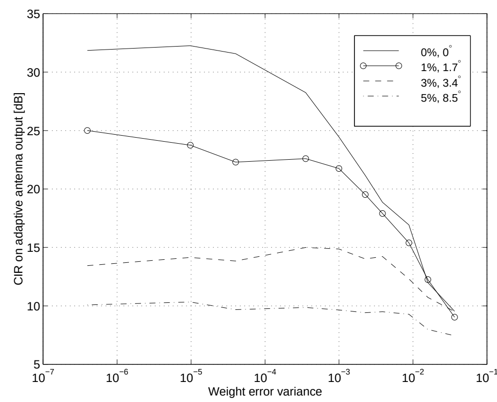

Figure 3: CIR_{out} as a function of weight quantisation error for different calibration errors, CIR_{in}=-15 dB, CNR=42 dB

tion errors when the CIR_{in} is varied by varying the interference to noise ratio (INR) is presented in Figure 4. The carrier to noise ratio (CNR) was held constant at 22dB. A comparison with the adaptive antenna testbed was made, and the CIR_{out} was measured for different calibration er-

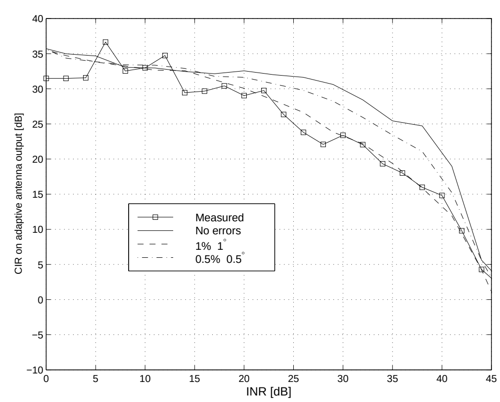

Figure 4: CIR_{out} as a function of interferer to noise ratio (INR) for different calibration errors. CNR=22 dB

rors and different CIR_{in}. The weight quantisation steps were set to 1 dB and 1°. The measured curve fits the curve with calibration error of 1% in relative magnitude and 1° in phase.

Furthermore, when INR$<$ 22 dB, i.e. when CIR$>$ 0dB, the calibration errors have a negligible effect on the antenna performance. When the interferer power gets large, the output noise is increasing, see equation (8) and the CIR decreases. When the total input power saturates the ADC, the CIR decreases abruptly and the antenna cannot maintain a CIR above 0 dB on the output.

5 Conclusion

It was found that the weight quantisation errors and the calibration errors increases the output noise power, thus decreasing the output CINR. The decrease is inversely proportional to the total impinging signal power on the array.

A simulation tool that model the hardware imperfection was developed, to study how the performance would improve if the calibration errors and weight errors was reduced. The simulation results showed good agreement with the measured results and the derived theoretical expression. The expression was derived by doing some simplifying assumptions, to get a quantitative expression for the adaptive antenna output CIR.

The system designer should put effort into making the phase and magnitude error variances equal to minimize a large overhead in either the phase-shifter or attenuator accuracy.

Furthermore, the balance between weight accuracy and calibration accuracy was examined, and it was shown that with a certain calibration accuracy, the output CINR could not improve above a certain limit, regardless of weight accuracy. Thus, these two sources of error should be balanced. For the testbed, it was seen that improving the calibration accuracy would gain more in output CIR as compared to improving the weight accuracy.

The results obtained in this work can be used to find out the impact of different implementation techniques on the total performance, to find performance bottle-necks and to help the designer to avoid over-dimensioning of antenna parts.

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