$\mathcal{H}_2$-optimal Anti-Windup Performance in SISO Control Systems

Jonas Öhr, [S] Stefan Rönnbäck [A] and Mikael Sternad [S]

S) Signals and Systems, Uppsala University, Sweden

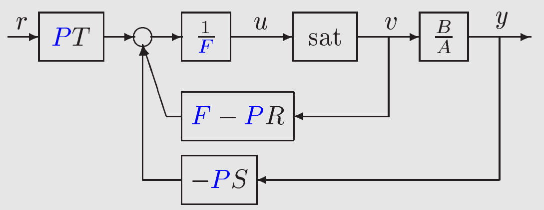
A) AssiDomän Corporate R & D, Piteå, Sweden

UPPSALA UNIVERSITET

AssiDomän

Abstract We consider a linear process, $y = \frac{B}{A}\mathrm{sat}[u]$, controlled by a feedback $Fu = (F - PR)\mathrm{sat}[u] - PSy + PTr$. Here, F, P are *anti-windup compensators*. We choose F as the nominal closed-loop characteristic polynomial $\alpha = RA + SB$ while P is obtained by a $\mathcal{H}_2$-optimization, where fast recovery after desaturation is traded against the stability margin by tuning a single parameter, ρ. We show that when tuning to attain a fast recovery after desaturation, this will *also* affect the instant of desaturation in a very desirable way.

System Structure, in polynomial form



- **controller with anti-windup**
  $Fu = (F - PR)\mathrm{sat}[u] - PSy + PTr$
- R, S, T **- nominal controller**
  $Ru = -Sy + Tr$ **or**
  $u = (1 - R)u - Sy + Tr$
- F, P **- anti-windup compensation polynomials,** see [1]

Tools for Analysis and Design

$\delta = \mathrm{sat}[u] - u = \frac{P\alpha}{FA}\mathrm{sat}[u] - \frac{PT}{F}r, \quad \alpha = RA + SB$

$y = y_i + y_\delta = \mathcal{H}_i r + \mathcal{H}_\delta \delta = \frac{BT}{\alpha}r + \frac{BF}{\alpha P}\delta$

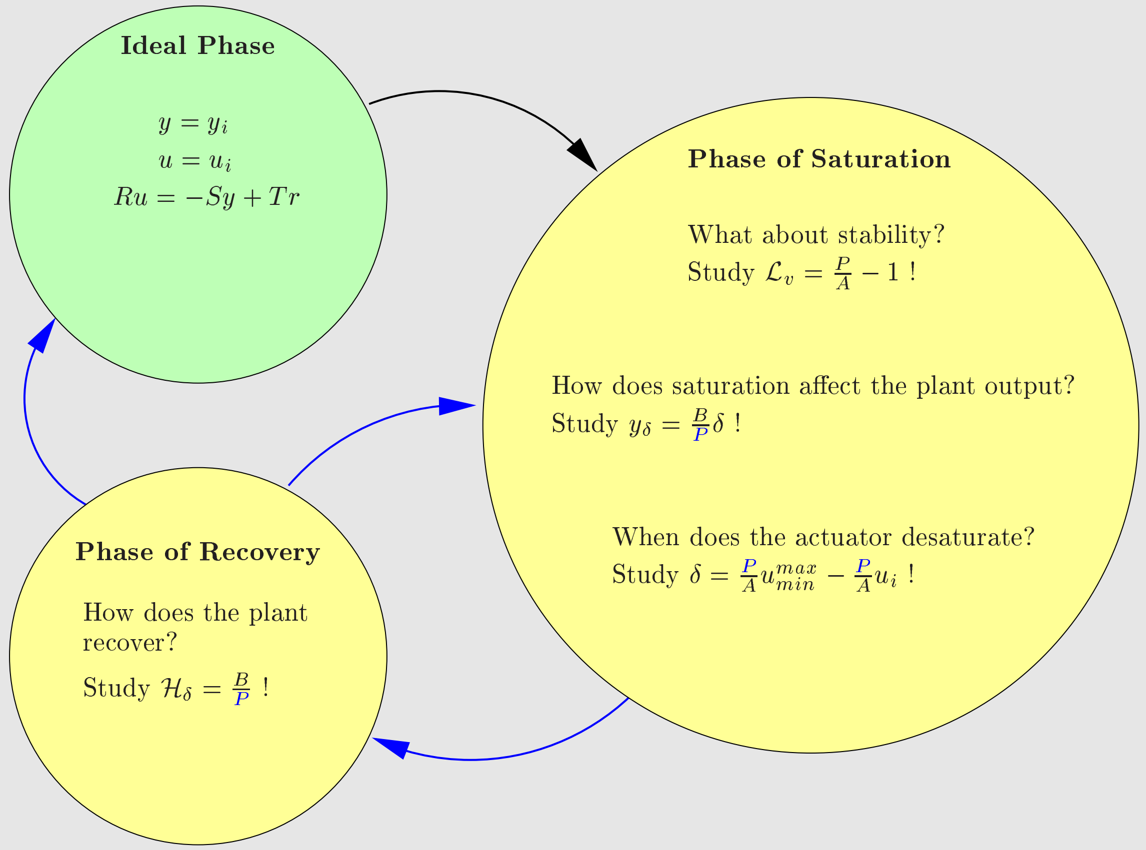
$\mathcal{L}_v = \frac{P\alpha}{FA} - 1$ - loop gain around $\mathrm{sat}[\cdot]$

The aim of the design is to obtain a fast recovery after desaturation ($\rho \to 0$), while preserving stability ($\rho \to \infty$), see [2]. In case of MIMO systems, see [3].

$$J = \|\mathcal{H}_\delta\|_2^2 + \rho\left\|(\mathcal{L}_v + 1)^{-1} - 1\right\|_2^2$$

Minimize J with respect to F and P by selecting

$$F = \alpha\,, \qquad rPP^* = BB^* + \rho AA^*$$



What happens when $\rho \to \infty$?

$$P \to A_{stab}, \qquad \mathcal{L}_v \to \frac{A_{stab}}{A} - 1$$

$$\delta \to \frac{A_{stab}}{A}\mathrm{sat}[u] - \frac{A_{stab}}{A}u_i$$

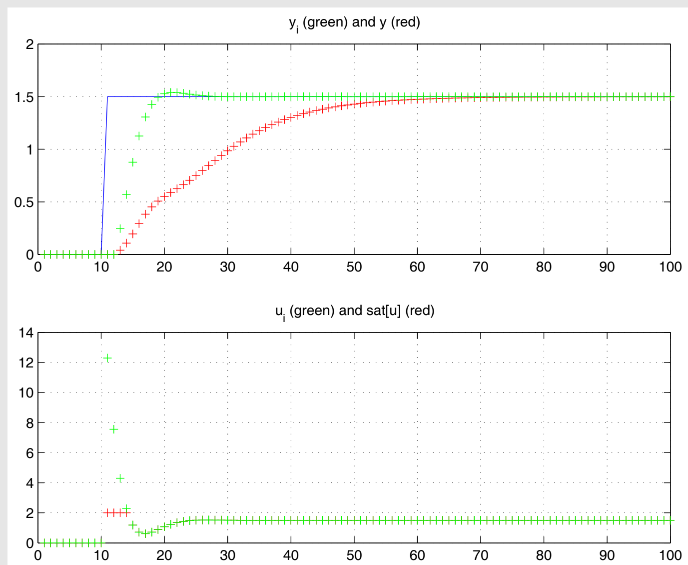
$$\mathcal{H}_\delta \to \frac{B}{A_{stab}}$$

Assume that A is stable. Then

- $\mathcal{L}_v \to 0$, which guarantees stability,
- $u = u_i$ which gives $\delta = \mathrm{sat}[u] - u_i$, and
- the plant recovers as $\frac{B}{A}$.

Example: Step response

The control signal, u, saturates ($\mathrm{sat}[u]$ red in the lower diagram) due to the step in r. Since $u = u_i$, (u_i, green in the lower diagram), u desaturates at the same time instant as u_i passes the u^{max} limit (in this case 2). The output recovers with the plant dynamics $\frac{B}{A}$.



What happens when $\rho \to 0$?

$$P \to \frac{q^k B_{stab}}{b_0}, \qquad \mathcal{L}_v \to \frac{q^k B_{stab}}{b_0 A} - 1$$

$$\delta \to \frac{q^k B_{stab}}{b_0 A}\mathrm{sat}[u] - \frac{q^k B_{stab}}{b_0 A}u_i$$

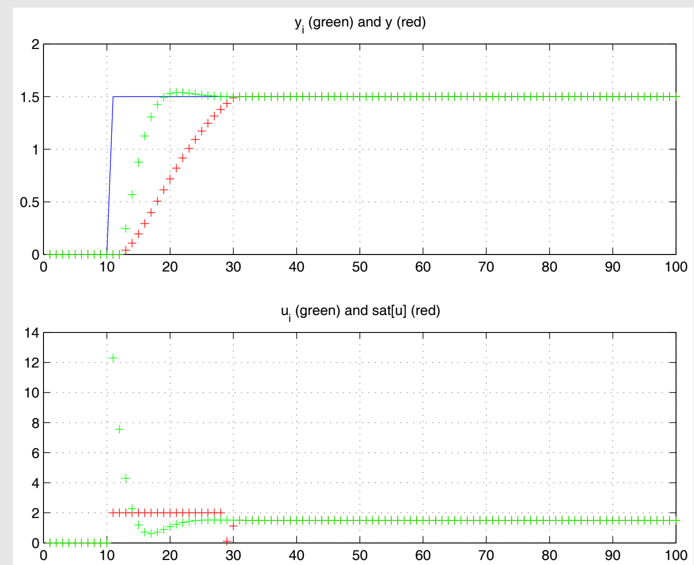
$$\mathcal{H}_\delta \to \frac{b_0 q^{-k} B}{B_{stab}}$$

Assume that B is “stable”. Then

- $\delta \to \frac{q^k}{b_0}(y - y_i)$, and
- the plant recovers as $b_0 q^{-k}$, (k-time delay in the plant).
- Stability is not guaranteed.

Example: Step response

The control signal, u, saturates ($\mathrm{sat}[u]$ red in the lower diagram) due to the step in r. Since $\delta = \frac{q^k}{b_0}(y - y_i)$, u will desaturate k *samples before the intersection* $y^{max} = y_i$ and y recovers completely within k samples after desaturation (in this case $k = 2$).



Conclutions

Tuning $\rho \to 0$ gives the most desirable performance, as long as stability is maintaned, and repeated re-saturations, which might occur after desaturation, are few. These undesired effects can be predicted by study a of $\mathcal{L}_v$ in a Nyquist plot.

References

[1] S. Rönnbäck, *Linear Control of Systems with Actuator Constraints.* PhD thesis, Division of Automatic Control, Luleå University of Technology, Sweden, 1993.

[2] M. Sternad and S. Rönnbäck, “A Frequency domain approach to anti-windup compensator design,” Inst. Technol., Uppsala Univ., Sweden, Rep. UPTEC 93024R, 1993.

[3] J. Öhr, Mikael Sternad and Anders Ahle'n*Anti-Windup Compensators for Multivariable Systems.* European Control Conferense, Brussels, Belgium, 1997, vol. 2, pp. FR-A G5.