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## ARRAY CHANNEL IDENTIFICATION USING DIRECTION OF ARRIVAL PARAMETRIZATION

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#### ABSTRACT

Identification of a radio channel to an antenna array, using a parametrization in direction of arrival and gains of signal paths, has been investigated. This channel estimate is compared to a least squares FIR channel identification. For the scenarios studied, the parametrized method had better performance than the least squares method. The parametrized method is however more complex and relies on estimates of the number of signals arriving to the antenna array.

#### 1. INTRODUCTION

In mobile radio communication, the use of an antenna array can improve the performance of the reception. By using an antenna array it becomes possible to form beams towards desired signals while "nulling out" interfering signals. A natural way to use the antenna array is to combine it with the intersymbol interference equalizer in the receiver. The equalizer used can, for instance, be a decision feedback equalizer (DFE) or a maximum likelihood sequence estimator (MLSE). It is assumed in this study, that the data is transmitted in bursts with a short training sequence in each burst used for for tuning the parameters of the equalizer/estimator. Here a DFE, designed in an indirect way, is used. The channel to the antenna array is first identified and the equalizer coefficients are computed using this estimated channel. In case of using a MLSE, the channel to the antenna array has to be identified in order to be able to compute the branch metrics.

Both for the DFE and the MLSE, it is of interest to get a good estimate of the channel. The straight forward way of achieving this estimate is to identify the channels to each of the antenna elements independently, as FIR channels, with a least squares algorithm. By doing this one does not utilize all of the structure in the incoming signal. If the signal arrives from a small number of directions, better channel estimates can be achieved if one parametrizes the joint channel to all the antenna elements in terms of directions of arrival (DOAs) and respective path gains. This can be realized by using the CDEML algorithm (Coherent DEcoupled Maximum Likelihood Estimation) [1]. A version of the CDEML algorithm is briefly presented and discussed below. The algorithm is also tested on a scenario with multipath propagation and intersymbol interference and compared with the traditional least squares channel identification. The performance of the algorithm is also measured in terms of the bit error rate (BER) of a DFE, designed from the identified channels.

#### 2. LEAST SQUARES CHANNEL IDENTIFICATION

The received signals at the M antenna elements,  $\mathbf{y}(t) = [y_1(t) \ y_2(t) \ \dots \ y_M(t)]^T$ , is modeled as

$$\mathbf{y}(t) = \mathbf{Bd}(t) + \mathbf{n}(t) \tag{1}$$

where  $\mathbf{d}(t) = [d(t) \ d(t-1) \dots d(t-nb)]^T$  and  $\mathbf{B}$  is the channel matrix of size  $M \times (nb+1)$ . The signal d(t) (=  $\pm 1$  here) is the transmitted symbol sequence. The noise plus interference is represented by the term  $\mathbf{n}(t)$ .

For each antenna element, m, we want to find the row m of  $\mathbf{B}$ ,  $\mathbf{b}_m$ , that minimizes the error between the estimated received signal, and the received signal over the training sequence  $(\{d(t), y_m(t)\}, t=1,2,...,N)$ :

$$e_m^2 = \sum_{t=nb+1}^{N} |y_m(t) - \mathbf{b}_m^T \mathbf{d}(t)|^2$$
 (2)

The row (i.e. FIR channel) that minimizes this norm can be found to be

$$\mathbf{b}_m = \hat{\mathbf{R}}_{\mathbf{d}y_m}^H \hat{\mathbf{R}}_{\mathbf{d}\mathbf{d}}^{-1} \tag{3}$$

where

$$\hat{\mathbf{R}}_{dy_m} = \frac{1}{N - nb} \sum_{t=n,b+1}^{N} \mathbf{d}(t) y_m^*(t) \tag{4}$$

and

$$\hat{\mathbf{R}}_{\mathbf{dd}} = \frac{1}{N - nb} \sum_{t=nb+1}^{N} \mathbf{d}(t) \mathbf{d}^{H}(t)$$
 (5)

The total, least squares estimated, channel matrix can be expressed as

$$\hat{\mathbf{B}}_{\mathrm{LS}} = \hat{\mathbf{R}}_{\mathrm{dv}}^{H} \hat{\mathbf{R}}_{\mathrm{dd}}^{-1} \tag{6}$$

where

$$\hat{\mathbf{R}}_{\mathbf{dy}} = \frac{1}{N - nb} \sum_{t=nb+1}^{N} \mathbf{d}(t) \mathbf{y}^{H}(t)$$
(7)

This channel estimate, as mentioned above, makes no assumption about the number of incoming signals. This makes it robust, but if the number of incoming signals per symbol delay is small, an improvement may be possible. A method utilizing this fact is presented next.

## 3. COHERENT DECOUPLED MAXIMUM LIKELIHOOD CHANNEL IDENTIFICATION

The method described here is a version of the CDEML algorithm presented in [1].

The overall channel model is the same as for the least squares identification

$$\mathbf{y}(t) = \mathbf{Bd}(t) + \mathbf{n}(t) \tag{8}$$

The noise plus interference,  $\mathbf{n}(t)$ , is here assumed to be circularly symmetric zero-mean Gaussian with second order moments

$$E[\mathbf{n}(t)\mathbf{n}^{H}(s)] = \mathbf{Q}\delta_{t,s} \quad E[\mathbf{n}(t)\mathbf{n}^{T}(s)] = \mathbf{0}$$
(9)

The matrix **B** is parametrized in terms of DOAs,  $\theta$ , for the incoming waves and their respective gains,  $\gamma$ :

$$\mathbf{B}(\theta, \gamma) = \mathbf{A}(\theta) \mathbf{\Gamma}(\gamma) \tag{10}$$

where

$$\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}_{01}(\theta_{01}) \dots \mathbf{a}_{0k_1}(\theta_{0k_1}) \dots \mathbf{a}_{nbk_{nb}}(\theta_{nbk_{nb}})] \quad (11)$$

and

The vectors,  $\mathbf{a}(\theta_i)$ , are the array response vectors for a signal arriving from angle  $\theta_i$ .

The parameter vectors  $\boldsymbol{\theta}$  and  $\boldsymbol{\gamma}$  are, as seen above, are partitioned according to which delay in B they correspond to:

$$\boldsymbol{\gamma} = [\gamma_0^T \ \gamma_1^T \ \dots \ \gamma_{nb}^T], \quad \boldsymbol{\gamma}_c = [\gamma_{c1} \ \dots \ \gamma_{ck_c}]^T$$
 (13)

$$\boldsymbol{\theta} = [\boldsymbol{\theta}_0^T \ \boldsymbol{\theta}_1^T \ \dots \ \boldsymbol{\theta}_{nb}^T], \quad \boldsymbol{\theta}_c = [\boldsymbol{\theta}_{c1} \ \dots \ \boldsymbol{\theta}_{ck_c}]^T$$
 (14)

It can be shown (see [1]) that a large sample maximum likelihood estimate of  $\theta$  and  $\gamma$  can be obtained by minimizing

$$F(\boldsymbol{\theta}, \boldsymbol{\gamma}) = tr[\mathbf{R}_{dd}(\mathbf{B}(\boldsymbol{\theta}, \boldsymbol{\gamma}) - \hat{\mathbf{B}}_{LS})^{H} \hat{\mathbf{Q}}^{-1}(\mathbf{B}(\boldsymbol{\theta}, \boldsymbol{\gamma}) - \hat{\mathbf{B}}_{LS})] (15)$$

where

$$\mathbf{R}_{\mathbf{dd}} = \lim_{N \to \infty} \frac{1}{N - nb} \sum_{t=nb+1}^{N} \mathbf{d}(t) \mathbf{d}^{H}(t), \tag{16}$$

 $\hat{\mathbf{B}}_{LS}$  is the least square channel matrix estimate and

$$\hat{\mathbf{Q}} = \frac{1}{N - nb} \sum_{t=nb+1}^{N} (\mathbf{y}(t) - \hat{\mathbf{B}}_{LS} \mathbf{d}(t)) (\mathbf{y}(t) - \hat{\mathbf{B}}_{LS} \mathbf{d}(t))^{H}$$
 (17)

is an estimate of the noise plus interference covariance matrix.

In digital communications the symbol sequence d(t) is assumed white. The covariance matrix,  $\mathbf{R}_{dd}$ , will then be diagonal. In this case, the maximum likelihood estimates of the angles,  $\theta$ , and the gains,  $\gamma$ , can be found by considering the following minimization for each column,  $\hat{\mathbf{b}}_c$ , in  $\hat{\mathbf{B}}$  separately:

$$\{\hat{\boldsymbol{\theta}_c}, \hat{\boldsymbol{\gamma}_c}\} = \arg\min_{\boldsymbol{\theta}_c, \boldsymbol{\gamma}_c} [\mathbf{A}(\boldsymbol{\theta}_c) \boldsymbol{\gamma}_c - \hat{\mathbf{b}}_c]^H \hat{\mathbf{Q}}^{-1} [\mathbf{A}(\boldsymbol{\theta}_c) \boldsymbol{\gamma}_c - \hat{\mathbf{b}}_c]$$
(18)

The fact that the minimization can be decoupled, greatly reduces the computational complexity. We can see from (18), that we are looking for  $\theta_c s$  and  $\gamma_c s$  that minimize, the weighted squared norm of the difference between the least squares identified column  $\mathbf{b}_c$  of  $\mathbf{B}_{LS}$  and the parametrized estimate  $\mathbf{A}(\boldsymbol{\theta}_c)\boldsymbol{\gamma}_c$ .

Returning to (15), we see that when  $\mathbf{R}_{dd}$  is diagonal, the overall criteria we want to minimize, is simply the weighted squared Frobenious norm of the difference between the parametrized channel matrix,  $\mathbf{B}(\boldsymbol{\theta}, \boldsymbol{\gamma})$  and the least squares estimated channel matrix  $\hat{\mathbf{B}}_{LS}$ , i.e.  $\parallel \mathbf{B}(\boldsymbol{\theta}, \boldsymbol{\gamma}) - \hat{\mathbf{B}}_{LS} \parallel_{\mathbf{Q}^{-1}}^{2}$ . The minimization of (18) with respect to  $\boldsymbol{\gamma}_{c}$  is:

$$\gamma_c(\boldsymbol{\theta}_c) = \{\hat{\mathbf{Q}}^{-1/2} \mathbf{A}(\boldsymbol{\theta}_c)\}^{\dagger} \hat{\mathbf{Q}}^{-1/2} \hat{\mathbf{b}}_c$$
 (19)

where  $(\cdot)^{\dagger}$  represents the Moore-Penrose pseudo-inverse.

By substituting this into the previous minimization we get

$$\hat{\boldsymbol{\theta}}_{c} = \arg \min_{\boldsymbol{\theta}_{c}} \{ \hat{\mathbf{b}}_{c}^{H} [\hat{\mathbf{Q}}^{-1} - \hat{\mathbf{Q}}^{-1} \mathbf{A} (\boldsymbol{\theta}_{k}) \times (\mathbf{A}^{H} (\boldsymbol{\theta}_{c}) \hat{\mathbf{Q}}^{-1} \mathbf{A} (\boldsymbol{\theta}_{c}))^{-1} \mathbf{A}^{H} (\boldsymbol{\theta}_{c}) \hat{\mathbf{Q}}^{-1} ] \hat{\mathbf{b}}_{c} \}$$
(20)

In order to assure convergence to the global minima, good initial values are required for the  $\theta$ s. In [1], it is proposed to initialize  $\theta_c$  with the  $k_c$  lowest local minima of the function

$$f(\theta) = \hat{\mathbf{b}}_c^H [\hat{\mathbf{Q}}^{-1} - \frac{\hat{\mathbf{Q}}^{-1} \mathbf{a}(\theta) \mathbf{a}^H(\theta) \hat{\mathbf{Q}}^{-1}}{\mathbf{a}(\theta)^H \hat{\mathbf{Q}}^{-1} \mathbf{a}(\theta)}] \hat{\mathbf{b}}_c$$
 (21)

This is exactly the cost function to be minimized if there was only one signal arriving per symbol delay. As the first term is independent of  $\theta$ , we can instead look for local maximus to the function

$$f_{0,c}(\theta) = \frac{\mathbf{a}^H(\theta)\hat{\mathbf{Q}}^{-1}\hat{\mathbf{b}}_c\hat{\mathbf{b}}_c^H\hat{\mathbf{Q}}^{-1}\mathbf{a}(\theta)}{\mathbf{a}(\theta)^H\hat{\mathbf{Q}}^{-1}\mathbf{a}(\theta)}$$
(22)

In the simulations performed in this study, this initialization procedure has been found to have some problems. It can for example have difficulties in estimating DOAs of signals that were close to a strong co-channel interferer. The presence of a strong co-channel interferer in the noise plus interference co-variance matrix,  $\mathbf{Q}$ , can cause a dip in the function  $f_{0,c}(\theta)$ .

In an attempt to alleviate this problem,  $Q^{-1}$ , can be removed from  $f_{0,c}(\theta)$ . The result is a simple "beamformer" DOA estimator. The initial values of the components of  $\theta_c$  can thus been chosen as the local maximas of the spectrum

$$f_{1,c}(\theta) = \frac{\mathbf{a}^H(\theta)\hat{\mathbf{b}}_c\hat{\mathbf{b}}_c^H\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\mathbf{a}(\theta)}$$
(23)

However, both of these methods have been found to have considerable difficulties in estimating initial values for the DOA:s when coherent sources are present. The peaks for the two functions in Equations 22 and 23 can then have peaks in directions not corresponding to a DOA. This is because side lobes of the "beamformers" involved may pick up the signals and combine them constructively depending on the particular relative phases of the signals involved. These combined signals can have a stronger amplitude then the signals caught by the main lobes of the "beamformers". A DOA will then be indicated at the wrong location.

Once the  $\theta_c$ s are estimated the  $\gamma_c$ s can be computed using

The estimated  $\hat{\theta}$  and  $\hat{\gamma}$  are used to form the improved parametrized channel matrix estimate

$$\hat{\mathbf{B}}_{\text{CDEML}} = \mathbf{A}(\hat{\boldsymbol{\theta}}) \mathbf{\Gamma}(\hat{\boldsymbol{\gamma}}) \tag{24}$$

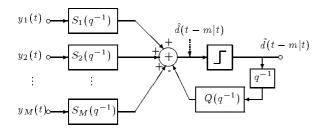


Figure 1: Structure of the general MISO FIR decision feedback equalizer, with M antenna elements. The FIR filters are represented as polynomials in the delay operator  $q^{-1}$ ,  $q^{-1}y(t) = y(t-1)$ .

# 4. INDIRECT, SPATIAL-ONLY INTERFERENCE CANCELLATION DECISION FEEDBACK EQUALIZER (IS-DFE)

The filter structure of a multiple input single output (MISO) DFE with FIR filters can be seen in Figure 1.

Place the equalizer parameters and the corresponding data samples in two vectors

$$\beta = [s_{01} \dots s_{0M} \ s_{10} \dots s_{nsM} \ Q_0 \dots Q_{nq}]^T$$
 (25)

$$\mathbf{Y}(t) = [ y_1(t) \dots y_M(t) \ y_2(t) \dots y_M(t - ns) d(t - m - 1) \dots d(t - m - 1 - nq) ]^T.$$
 (26)

The parameter vector  $\boldsymbol{\beta}$ , that minimizes the expected mean square error in  $\hat{d}(t-m|t)$  is given by

$$\beta = (\mathbf{R}_{\mathbf{YY}}^{-1} \mathbf{R}_{\mathbf{Yd}})^* \tag{27}$$

where

$$\mathbf{R}_{\mathbf{YY}} = E[\mathbf{Y}(t)\mathbf{Y}^{H}(t)] \quad \mathbf{R}_{\mathbf{Yd}} = E[\mathbf{Y}(t)d^{H}(t-m)] \quad (28)$$

and  $(\cdot)^*$  represents elementwise complex conjugation.

In the indirect design of the DFE, the channel to the antenna elements,  $\mathbf{B}$ , is first identified. The residues,  $\hat{\mathbf{n}}(t)$ , at the antenna elements are then computed as

$$\hat{\mathbf{n}}(t) = \mathbf{y}(t) - \hat{\mathbf{B}}(q^{-1})\mathbf{d}(t)$$
(29)

An estimate of  $\beta$  is computed as

$$\hat{\boldsymbol{\beta}} = (\hat{\mathbf{R}}_{\mathbf{Y}\mathbf{Y}}^{-1}\hat{\mathbf{R}}_{\mathbf{Y}\mathbf{d}})^* \tag{30}$$

where

$$\hat{\mathbf{R}}_{\mathbf{YY}} = \frac{1}{N - nb} \sum_{t=nb+1}^{N} \mathbf{Y}_{\hat{s}}(t) \mathbf{Y}_{\hat{s}}^{H}(t) + \hat{\mathcal{R}}_{\hat{n}\hat{n}}$$
(31)

with

$$\mathbf{Y}_{\hat{s}}(t) = [\hat{s}_1(t) \dots \hat{s}_M(t) \ \hat{s}_2(t) \dots \hat{s}_M(t - ns) \\ d(t - m - 1) \dots d(t - m - 1 - nq)]^T.$$
 (32)

and

$$\hat{s}_i(t) = \hat{B}_i(q^{-1})d(t) \tag{33}$$

The matrix  $\hat{\mathbf{R}}_{\hat{n}\hat{n}}$  is a block diagonal matrix with the matrix  $\hat{\mathbf{R}}_{\hat{n}\hat{n}}$  along the diagonal, except for an all zero last  $nq+1\times nq+1$  block. The matrix  $\hat{\mathbf{R}}_{\hat{n}\hat{n}}$  is the *spatial* correlation matrix for the residuals from the least squares estimation.

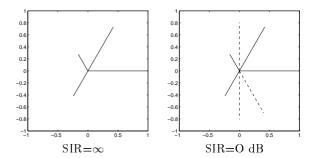


Figure 2: Signal configuration. The solid lines represents the incoming directions of the desired signal and the dashed lines represent co-channel interferers. The left figure depicts the case without co-channel interferers and the right figure depicts the case with co-channel interferers.

The first term in  $\hat{\mathbf{R}}_{\mathbf{YY}}$  is the an estimate of the part of  $\mathbf{R}_{\mathbf{YY}}$  that corresponds to the desired signal. The second term in is the an estimate of the part that corresponds to the noise plus interference. Only the spatial color have been considered in this latter term. The second factor in (30),  $\hat{\mathbf{R}}_{\mathbf{Yd}}$ , is estimated as

$$\hat{\mathbf{R}}_{\mathbf{Yd}} = \hat{\mathbf{b}}_{m+1} \tag{34}$$

where  $\hat{\mathbf{b}}_{m+1}$  is the (m+1):th column of  $\hat{\mathbf{B}}$ , corresponding to the tap with delay m in the channel.

By only considering the spatial color of the residuals, a full rank estimate of  $\mathbf{R}_{YY}$  is achieved. The effect on the resulting DFE, is that it will only cancel interference by spatial nulling, i.e. no temporal interference suppression. The DFE is therefor called an IS-DFE (Indirect DFE with Spatial only interference cancellation), see [2].

#### 5. SIMULATION STUDY

A simulation study has been conducted in order to evaluate the performance of the algorithm in a scenario with multipath propagation and intersymbol interference.

#### 5.1. Scenarios

The algorithms were tested using a circular array, see Figure 3. The desired signal arrives from the directions 0.60,-120 and 120 degrees. The respective channels are  $1+0.5q^{-1}$ ,  $0.5q^{-1}+0.8q^{-2}$ ,  $0.5q^{-2}+0.2q^{-3}$  and  $0.2q^{-3}+0.3q^{-4}$ . Two-tap channels are chosen in order to simulate imperfect sampling timing or partial response modulation. When co-channel interferers are included they impinge on the antenna array through single tap channels from the directions 90, -60 and -90 degrees, with a total average SIR of 0 dB. The SNR was varied from -3 dB to 6 dB. The scenario is illustrated in Figure 2. The number of training symbols used were 26.

An FIR channel model with 5 taps was used for the channel matrix. The DFE used had feedforward filters of length 4, feedback filters of length 3 and a smoothing lag, m, of 3.

#### 5.2. Initial values for the DOAs

Both the initialization suggested in [1] and the initialization in (23) have problems with achieving good initial values for the DOAs. In Figure 4, examples of the two functions can be seen. By comparing the plots for the functions  $f_{0,c}(\theta)$  and  $f_{1,c}(\theta)$ , one can see the result of including  $Q^{-1}$  in  $f_{0,c}(\theta)$ . It can be seen that

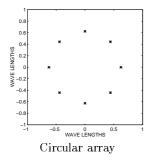


Figure 3: Antenna array configuration.

 $f_{0,c}(\theta)$  has dips or reduced amplitude at or close to the locations of the co-channel interferers.

Both initializations find the DOA for the signal with delay 0. In the case with delay 1 neither of the initialization algorithms give a good initial estimate.

For the cases with delay 2 and delay 3, the function  $f_0$  has the peak corresponding to the signal arriving from -60 degrees reduced as a result of the presence of the cochannel interferer at the same angle. Note that the co-channel interferer at -45 degrees has been moved to -60 degrees in order to illustrate the masking effect. As a result the algorithm using  $f_0$  misses this DOA.

When the signal strength becomes low, as for delay 4. The peaks corresponding to the true DOA:s becomes less distinctive and more difficult to detect. Both algorithms suffer from this.

In conclusion it can be said that neither of the functions are ideal for finding initial values for the DOAs. In the experiments performed in conjunction to this study, the DOA estimator  $f_{1,c}(\theta)$ , have been slightly better for estimating the DOAs. This estimator has therefor been used in the simulations in this study.

If a uniform linear array is used, the  $k_c$ -dimensional search in (20) can be reduced to a polynomial root-finding operation using a technique similar to that developed in [3] and [4]. Considering the problems with finding good initial values for the DOA:s it is likely a good idea to restrict the array geometry to the ULA case, and use the root-finding approach.

### 5.3. Performance in terms of channel estimation and IS-DFE bit error rate.

The CDEML algorithm was evaluated assuming different number of DOAs, initializing them either with the true DOAs, or estimating initial DOAs using the spectrum in (23). When using the "true" DOAs as initial values, if the number of DOAs assumed where greater than the true number of DOAs, the extra DOAs where randomized.

The total mean square error in the estimated channel for different signal-to-noise ratios can be seen in Figure 5. As can be seen, the best performance is achieved using the CDEML algorithm with the correct number of DOAs, and the true DOAs as initial values. It can be seen that both overestimating the number of signal paths and using estimated DOAs as initial values, deteriorates the performance of the algorithm. In the scenarios studied, however, all the CDEML versions had better or equal performance, than the least squares identification.

It should be kept in mind though, that the scenario investigated is very well suited for a DOA parametrized channel identification as CDEML. This is because many antenna elements were used, and only a few signal paths per delay impinges on the antenna array. The number of degrees of freedom is thus considerably reduced by parametrizing the channel in DOAs, rather

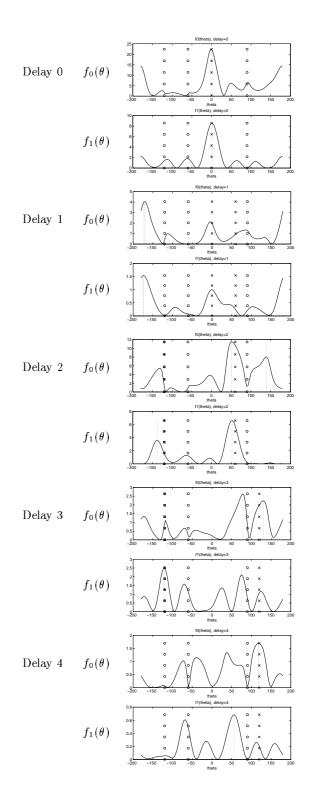


Figure 4: Functions used for finding initial values for the DOAs. f0(theta) is the function in (21) and f1(theta) is the function in (22). The functions are plotted for the delays 0,2 and 3 in the channel for the ULA case. The minimas chosen as initial values are marked with dotted vertical lines. The true DOAs are marked with crosses. The directions of the co-channel interferers are marked with lines of circles. SIR=0dB and SNR=3dB. Note that the co-channel interferer at -45 degrees has been moved to -60 degrees in order to illustrate the masking effect as discussed in the text.

than using the full channel matrix representation.

The improved channel estimates reflect on the improved bit error rates as seen in Figure 5. In these figures, the BER for a DFE where the channel taps are tuned directly using 1000 training symbols, is also presented. It will not be possible to achieve BERs, using a DFE of the specified order, that are much lower than this curve. Note the improvement in the case with an SIR of 0 dB. It appears that, at least in this simulation, the CDEML algorithm handles the presence of co-channel interferers better than the least squares identification.

#### 6. CONCLUSIONS

It has been shown that when using an antenna array it is possible to improve the channel identification to the array by parametrizing the channel into directions of arrival of the signal paths, and their gains. The CDEML algorithm performs this in a computationally efficient way as the problem is decoupled into one minimization problem per tap in the channel. In the simulations performed in this study, it was found that the CDEML algorithm handles the presence of co-channel interferes better than the least squares algorithm.

As seen from the simulations, non-negligible improvements can be achieved in the CDEML algorithm by using the correct or near correct number of paths per delay. It should therefor be of importance to use algorithms that can estimate the number of DOAs present. It has also been shown that it is of importance to have good initial estimates of the DOAs.

Unfortunately, neither the method suggested in [1] or the simplified version presented in (23), does a good job in forming initial estimates of the DOAs. Unless better algorithms are used for finding initial estimates of the DOAs, the best strategy is probably to constrict the array geometry to a uniform linear array, and replace the nonlinear minimization with a polynomial root-finding technique similar to the one in [3] and [4].

Although the CDEML algorithm had better performance than the algorithm based on the least squares identification, it should be noted that there is a non-negligible increase in the complexity and the CDEML algorithm will also be less robust against missmodeling of the scenario.

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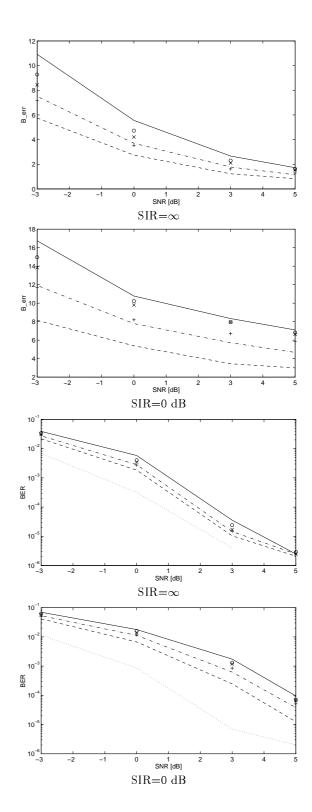


Figure 5: Total mean square error in the channel matrix, B\_err, and BER for the equalizer. Least squares identification (solid line), CDEML with true number of DOAs and true DOAs as initial values (dashed), CDEML with one extra DOA per delay and true DOAs as initial values (dash-dotted), CDEML with true number of DOAs and estimated DOAs as initial values (+), CDEML with one extra DOA per delay and estimated DOAs as initial values (x), CDEML with four DOAs for each delay and estimated DOAs as initial values (o).