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Spatio-Temporal Equalization for Multipath Environments in Mobile Radio Applications*

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Abstract — Combined spatial and temporal equalization using an antenna array combined with a decision feedback equalization scheme is investigated. In particular a TDMA type system with a relatively short training sequence is considered. Three algorithms are introduced. The first two algorithms are based on indirect schemes, where the channels to each receiver antenna element are identified. The identified channels and the correlations of the residuals are then used for the tuning of the beamformer/equalizer coefficients. The spatio-temporal correlations of the residuals are used in the first algorithm while in the second algorithm only the spatial correlations of the residuals are considered. The third algorithm forms a number of beams by using mixtures of different delayed versions of the training sequence as reference signals. It then performs temporal equalization by combining the outputs from the different beamformers, with appropriate delays. This latter algorithm requires less computations for the tuning of the equalizer, at the expense of a performance degradation in general. The algorithms are evaluated with simulations of multipath scenarios involving co-channel interference.

I. Introduction*

A motivation for the methods presented in this paper is the expansion of digital mobile radio communications. A TDMA type mobile radio system is considered. Here, data are transmitted in bursts, and attached to each burst is a training sequence of short duration. The channel is characterized by multipath propagation and co-channel interference. Therefore the received signal needs to be processed, in order to retrieve the transmitted message.

One way of doing this is to use a decision feedback equalizer (DFE). A feedforward filter then filters the received signals while feedback filter processes previously received symbols and cancels their impact on the output of the feedforward filter. The feedforward and feedback filtered signals are combined and fed into a decision device, which makes decisions on a symbol by symbol basis. This is an example of *temporal equalization*.

In the case of multipath propagation it is reasonable to expect that signal paths with relative time delay differences larger than or equal to one symbol period, T, will impinge on the receiver antenna from different angles. By using filtering in the spatial domain, an antenna array can separate different signal paths. We can then either use only one of the signal paths, the strongest one, or we can use a combination of all the signal paths. We regard this as an example of *spatial equalization*.

In [1], an algorithm is proposed which uses an LMS adaptive array to train sets of weights to optimally receive a reference signal with different delays. The outputs, corresponding to the different sets of weights, are then delayed, weighted and summed. It is proposed to choose the weighting coefficients so that maximum ratio combining is achieved. This combining is, however, not optimal if the noise is correlated between signals to be combined. The algorithm also lacks a decision feedback, which could improve the equalizing capability.

In [2], a structure is proposed that combines *one* antenna array beamformer with *one* FIR-filter. The signals from the antennas are combined in a weighted sum and then passed through the FIR-filter. Two algorithms are proposed for adaptation of the antenna weights and filter coefficients. Implicitly, the algorithm utilizes delayed desired signals but it has no decision feedback filter.

A reduced-complexity multichannel DFE is proposed in [3]. P sets of beamforming weights are connected to the antennas. Each beamformer output is then fed into a feedforward FIR-filter. The outputs from these filters are then summed and old decisions, filtered through a common FIR feedback filter, is subtracted. Symbol decisions are then formed based on the resulting signal. The beamformer and filter coefficients are adapted simultaneously. This algorithm has a quite general structure and can be used with different levels of complexity.

In [4], an algorithm is proposed which forms several beams in order to receive the training sequence with different delays. The resulting signals and filtered old decided symbols are then weighted and summed, in order to minimize the mean square error between the equalizer output and the training sequence. A drawback with this algorithm is that it suffers from a performance degradation if multiple delayed versions of the training sequence are present in the signal arriving from a given direction. This will typically be the case if the modulation technique introduces intersymbol interference.

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In this paper we will consider combined spatial and temporal equalization, by means of an antenna array and a DFE equalization scheme. The DFE utilizes filter of finite impulse response type (FIR). It is of multiple-input-single-output type (MISO), see Figure 1.

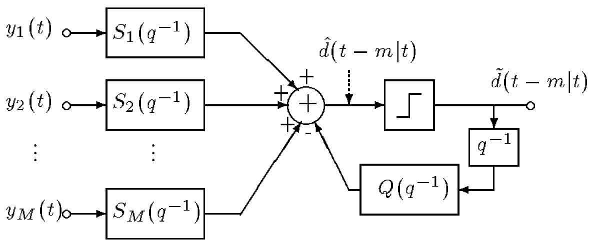

Figure 1: Structure of the general MISO FIR decision feedback equalizer, with M antenna elements.

The FIR filters $S_i(q^{-1}) = s_{0i} + s_{1i}q^{-1} + \ldots + s_{nsi}q^{-ns}$ and $Q(q^{-1}) = Q_0 + Q_1 q^{-1} + \ldots + Q_{nq}q^{-nq}$ are represented as polynomials in the unit delay operator q^{-1} ($q^{-1}y(t) = y(t-1)$), with complex coefficients. The number m is the smoothing-lag used in the estimation. The feedforward filters, $S_i(q^{-1})$, filter the received sampled baseband signals, $y_i(t)$, from the antenna elements. Their outputs are summed and the output of the common feedback filter, $Q(q^{-1})$, is subtracted. The resulting signal, $\hat{d}(t-m|t)$, is then fed into the decision device, to form the symbol estimate $\tilde{d}(t-m|t)$.

A possible disadvantage with the general structure depicted in Figure 1, is that if all the coefficients are to be optimized jointly (for example as in the D-DFE algorithm below), this may require many arithmetic operations. For this reason, it is of interest to investigate algorithms with lower computational complexity.

We propose three algorithms with filter structures as in Figure 1. The first two algorithms are indirect methods, tuning the equalizer coefficients partly by using estimated channels. The third algorithm is an improvement of the algorithm proposed in [4]. We compare with an algorithm with direct tuning of the coefficients of the general MISO FIR decision feedback equalizer and also to the algorithm proposed in [4]. The algorithms are evaluated for two different scenarios.

II. Algorithms

We assume that data is transmitted in bursts, in which the first N symbols constitute a known training sequence, $d(t)$, t=1,2,...,N and the remaining symbols are unknown data. The training sequence is used to tune the equalizer parameters. The so obtained equalizer estimates the remaining data of the burst. The transmitted symbols, $d(\cdot)$, are assumed binary, with values 1 or -1. The algorithms to be evaluated are presented next.

Directly tuned Decision Feedback Equalizer (D-DFE): This is the general MISO FIR decision feedback equalizer, with direct tuning of the coefficients over the training data. For a given order of the filters, the coefficients of the equalizer, shown in Figure 1, are chosen to minimize $\sum_{t=m+nq+2}^{N}(\hat{d}(t-m|t) - d(t-m))^2$.

Define the coefficient parameter vector, θ, and the data vector, $Y(t)$, as

$$\theta = [\ s_{01}\ \ldots\ s_{ns1}\ \ s_{02}\ \ldots\ s_{nsM}\ \ Q_0\ \ldots\ Q_{nq}\ ]^T \qquad (1)$$

$$\begin{aligned} Y(t) = [\ & y_1(t)\ \ldots\ y_1(t-ns)\ \ y_2(t)\ \ldots\ y_M(t-ns) \\ & d(t-m-1)\ \ldots\ d(t-m-1-nq)\ ]^T. \end{aligned} \qquad (2)$$

The equalizer coefficients are computed as

$$\hat{\theta} = \left(\hat{R}_{YY}^{-1}\hat{R}_{Yd}\right)^* \qquad (3)$$

where $(\cdot)^*$ represents elementwise complex conjugation and

$$\hat{R}_{YY} = \frac{1}{(N-t_0+1)}\left(\sum_{t=t_0}^{N} Y(t)Y^H(t)\right) + \sigma_a^2 I_{yy} \qquad (4)$$

$$\hat{R}_{Yd} = \frac{1}{(N-t_0+1)}\sum_{t=t_0}^{N} Y(t)d^H(t-m) \qquad (5)$$

with $t_0 = m + nq + 2$. The matrix I_{yy} has an $M(ns+1) \times M(ns+1)$ unit matrix in its the upper left corner and zeros elsewhere. The term $\sigma_a^2 I_{yy}$ is added in order to regularize the matrix $\hat{R}_{YY}$. Asymptotically, with infinite data (and some noise), σ_a^H should be set to zero. However, with a finite amount of data and/or a limited computational accuracy, the solution can be improved by a proper choice of σ_a^2. The matrix $\hat{R}_{YY}$ cannot be inverted without this regularization if the number of training symbols are fewer than the number of parameters.

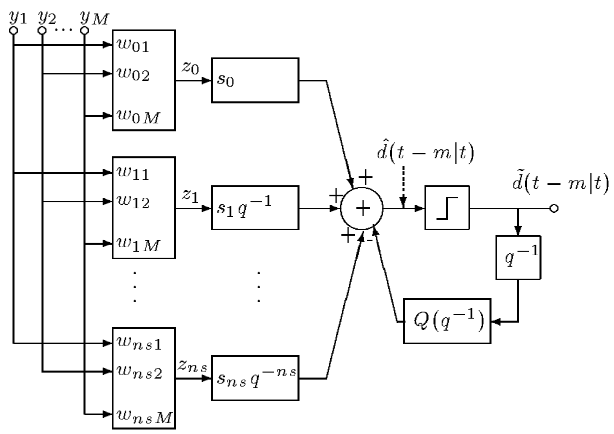

Figure 2: Structure of the MIB- and MIMB-DFE:s

Indirectly tuned DFE (I-DFE): The training sequence can be utilized for identifying the FIR channels between the transmitted symbols and the received samples at the antenna elements. This identification is performed by applying the least squares method to each antenna signal separately. The identified channels are used to estimate the spatio-temporal correlations of the desired signals. The residuals are used for constructing an estimate of the spatio-temporal correlations of the noise plus interference. These two estimates are then combined into an estimate, $\hat{R}_{YY}$. The estimate $\hat{R}_{Yd}$ is formed from the identified channels. The matrix $\hat{R}_{YY}$ is regularized as in equation (4). Also in this case regularization is required if the training symbols are fewer than the parameters.

Indirectly tuned DFE with spatial-only interference cancellation (IS-DFE): The estimate of $\hat{R}_{YY}$ can be improved by taking only the spatial correlations of the residuals into consideration. Let

$$XS\hat{B}_i(q^{-1}) = \hat{b}_{0i} + \hat{b}_{1i}q^{-1} + \ldots + \hat{b}_{nbi}q^{-nb} \tag{6}$$

be the estimated channel to antenna i. Thus,

$$y_i(t) = \hat{s}_i(t) + \hat{n}_i(t) = \hat{B}_i(q^{-1})d(t) + \hat{n}_i(t) \tag{7}$$

for $t = 1 + nb, \ldots, N$, where $\hat{s}_i(t)$ is an estimate of the desired (disturbance-free) received signal, while the residual $\hat{n}_i(t)$ captures noise, interference and model errors. If $\hat{n}_i(t)$ is assumed uncorrelated with $\hat{s}_j(t-\tau)$, for all j and τ, the estimate (3) can then be computed using

$\hat{R}_{Yd} = [\ \hat{b}_{m1} \ldots \hat{b}_{(m-ns)M}\ 0 \ldots 0\ ]^T$; $\hat{R}_{YY} = \hat{R}_{Y_sY_s} + \hat{R}_{Y_nY_n}$

where Y_s is formed by substituting $\hat{s}_i(t)$ for $y_i(t)$ in $Y(t)$ in (2) and where

$$\hat{R}_{Y_nY_n} = \frac{1}{N-nb}\sum_{t=1+nb}^{N} \mathcal{N}(t)\mathcal{N}^H(t) \tag{8}$$

with $\mathcal{N}(t) \triangleq [\hat{n}_1(t)\mathbf{I}_{ns+1} \ldots \hat{n}_M(t)\mathbf{I}_{ns+1}\ \mathbf{0}\ ]$. Fewer parameters are estimated when considering only the spatial correlations of the noise plus interferers, as compared to estimating the spatio-temporal correlations. This estimate will then, of course, not contain any information about the temporal correlations of the noise plus interferers. However, if we are primarily interested in nulling out interferers in the spatial domain, this is of secondary importance. In this case regularization was not used. The matrix $\hat{R}_{YY}$ will here normally have full rank for training sequence lengths larger than the number of antennas.

Multiple Independent Beam Decision Feedback Equalizer (MIB-DFE): This is the algorithm proposed in [4]. The MIB-DFE combines an antenna weight adaptation algorithm with a DFE scheme. The structure of the equalizer is depicted in Figure 2.

The MIB-DFE has $ns+1$ sets of M antenna weights, w_{ij}. First, the $ns+1$ sets of antenna weights are chosen to minimize $\sum_{t=ns-i+1}^{N}(z_i(t) - d(t-(ns-i)))^2$, i=0,1,...,ns, respectively. This means that each set forms a beam in order to optimally receive versions of the training sequence with different delays. The $ns+1$ output signals, $z_i(t)$, i=0,1,...,ns, from the antenna weight sets are then computed over the duration of the training sequence. Second, the DFE filter coefficients s_0,s_1,...,s_{ns} and Q_0,Q_1,...,Q_{nq} are computed to minimize $\sum_{t=m+nq+2}^{N}(\hat{d}(t-m|t) - d(t-m))^2$ using signals $z_i(t)$ and $d(t-m-1)$ as inputs.

Multiple Independent Mixed Beam Decision Feedback Equalizer (MIMB-DFE): We here add an extra coefficient c_i in the MIB-DFE, for each set of antenna weights. The antenna weights, w_{ij}, and the delay weights, c_i, are chosen to minimize $\sum_{t=ns-i+1}^{N}(z_i(t) - c_i d(t-1-(ns-i)) - d(t-(ns-i)))^2$, i=0,1,...,ns, respectively. Each set forms a beam and a mixture of $d(t-(ns-i))$ with $d(t-1-(ns-i))$, in order to minimize the MSE between the training signal $d(t-(ns-i))$ and the output of the beamformer subtracted with $c_i d(t-1-(ns-i))$. The coefficients for the temporal equalization are subsequently tuned as for the MIB-DFE.

The motivation for this algorithm is that when a mixture of two adjacent symbols is impinging on the array from each direction, we do not want to treat one of them as desired signal and the other one as a disturbance. This could cause the gain in that direction to be reduced. If, instead, we treat both signals as desired signals, we potentially obtain a higher gain in the relevant direction. If more than two symbols are involved in the mixture impinging from each direction, then extra coefficients, similar to c_i, can be included in the minimization procedure.

Algorithms similar to the ones proposed in [3] have also been studied. We do not include them in the present study, as we have not succeeded in making them perform well enough on the scenarios and the lengths of the training sequences considered.

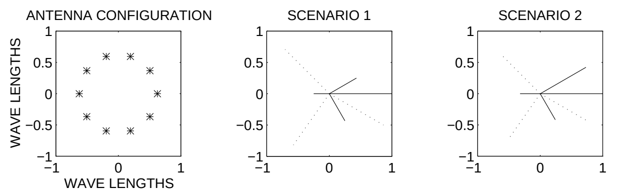

Figure 3: Left figure: Antenna configuration. Middle and right figure: Scenario 1 and 2. Desired signals (solid) and co-channel interferers (dotted). The line lengths are proportional to the square root of the power impinging from each direction. The antenna is located at the origin.

III. Simulation results

The algorithms were tested on two different scenarios, described below. In all simulations, a circular array consisting of ten antennas, as shown in Figure 3, was used. Parameter values ns=3, nq=2 and a smoothing lag of m=3, were used in all algorithms. The regularizing constant σ_a^2 was set equal to the variance of the noise. This value was found to work well for our simulations. Note, however, that for long training sequences, compared to the number of parameters that are tuned jointly, this value should likely be decreased. How to choose σ_a^2 if the noise variance is unknown has not been investigated.

Scenario 1: The desired signal is impinging on the array from the directions $\alpha = 0$, 30 -60 and 180 degrees, through the channels $B(q^{-1}) = 1$, $0.5q^{-1}$, $0.5q^{-2}$ and $-0.25q^{-3}$ respectively. Three co-channel interferers are impinging on the array from the directions $\alpha_{co} = 135$, -30 and 235 degrees respectively, each having a constant channel $B_{co}(q^{-1}) = b_{co}$. The constant b_{co} was in each scenario selected such that the SIR, averaged over the antenna elements, became 0 dB. Independent white noise giving a SNR of 3 dB, averaged over the antenna elements, was also added. See Figure 3.

Scenario 2: The desired signal is impinging on the array from the same directions as in Scenario 2, but now each channel contains a mixture of two adjacent symbols. The respective channels are $B(q^{-1}) = 1+0.5q^{-1}$, $0.5q^{-1}+0.8q^{-2}$, $0.5q^{-2}+0.2q^{-3}$ and $0.2q^{-3}+0.3q^{-4}$. Co-channel interferers and noise enter as in Scenario 1. See Figure 3.

A motivation for Scenario 2 is that it may be impossible to sample in such a way that each sample contains only one symbol. In for instance the GSM system, where a GMSK modulation with a BT-product of 0.3 is used, Scenario 1 will never occur. From each direction a partly unknown mixture of two to three delayed versions of the training sequence will arrive, due to intersymbol interference introduced by the partial response modulation. Scenario 2 is then a more realistic model.

For each scenario and for *different lengths of the training sequence*, 200 experiments with different noise and co-channel interferer realizations were conducted. In each experiment, the equalizer parameters were computed based on the training sequence. Then the bit error rate (BER) was estimated over a data sequence with a length selected such that typically 100 bit errors were generated for each data point, in total. The resulting BER is summarized for the two scenarios in Figure 4.

In both scenarios the IS-DFE has the best performance and the I-DFE has a better performance than the D-DFE, in terms of BER. The MIMB-DFE has worse performance than the IS-DFE. The MIMB-DFE is however, except for the very short training sequence lengths, comparable to the I-DFE in Scenario 1 and slightly better in Scenario 2. The MIB-DFE has a reasonably good performance for Scenario 1. However, when, as in Scenario 2, also a delayed training sequence is arriving from each direction, then the MIB-DFE suffers from a performance degradation. The MIMB-DFE handles the situation in Scenario 2 better, as it is designed to do.

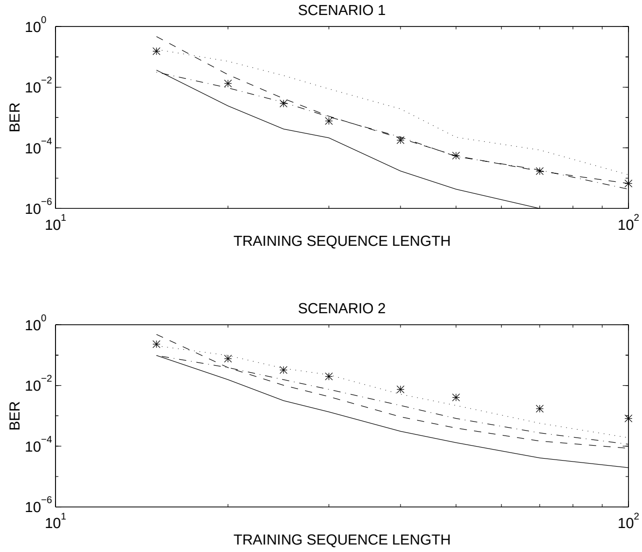

Figure 4: BER for Scenario 1 and 2: D-DFE (dotted), I-DFE (dash-dotted), IS-DFE (solid), MIB-DFE (star) and the MIMB-DFE(dashed).

IV. Computational complexity

The D- and the I-DFE:s require the largest amount of computations for their tuning. The IS-DFE requires less computations and the MIMB-DFE requires the least number of computations for the tuning.

For relative comparisons, we here define a complexity unit, *cu*. One *cu* is defined to be the amount of computations required for one complex multiplication. We assume that a *real* division requires 18 time as long time as a *real* multiplication†. A *complex* division will then correspond to 6.5 complex multiplications, and is counted as 6.5 *cu*. Additions are neglected in the complexity estimations, but their number is roughly be proportional to the number of multiplications (which dominate). The computational complexity, C, of the D-, I-, IS- and the MIMB-DFE:s are approximately given by

$$C_{\text{D-DFE}} \approx \frac{1}{6}(M(ns+1)+nq+1)^3 + \frac{1}{2}(M(ns+1)+nq+1)^2(N-m-nq+\frac{11}{2})\ cu \quad (9)$$

$$C_{\text{I-DFE}} \approx \frac{1}{6}(M(ns+1)+nq+1)^3 + \frac{1}{2}M^2(ns+1)^2 + \frac{19}{4}(M(ns+1)+nq+1)^2 + (2N-nb)(nb+1)M + \frac{1}{2}(nb+1-\frac{ns}{2})(ns+1)^2M^2\ cu \quad (10)$$

$$C_{\text{IS-DFE}} \approx C_{\text{I-DFE}} - \frac{1}{2}M^2((ns+1)^2-1)N\ cu \quad (11)$$

†This is the case for the signal processor TMS320C50

and

$$C_{\text{MIMB-DFE}} \approx \frac{1}{6}(M+1)^3 + \frac{1}{6}(ns+nq+2)^3 + (M+1)^2(\frac{1}{2}N + ns + \frac{19}{4}) + 2(M+1)N(ns+1) + \frac{1}{2}(ns+nq+2)^2(N-ns+\frac{19}{2}) + (ns+nq+2)(N-ns)\ cu \quad (12)$$

respectively. The positive definite and hermitian structure of the involved matrices has been taken into account.

Added to these complexities should be the computations when applying the equalizers to the data sequence

$$C_{\text{execution}} = (M*(ns+1)+nq+1)*N_{data}\ cu \quad (13)$$

As a numerical example, the computational complexities can be evaluated for the case in the simulations with $M = 10$, $ns = 3$, $nq = 2$ and $nb = 4$. The number of training symbols are set to 26 and the number of data symbols are set to 116 (as in the GSM system). The approximative total complexities of the algorithms with these parameters can be seen in Table 1, also for M=5,20.

	Approx. Complexity [*cu*]		
Algorithm	$M = 5$	$M = 10$	$M = 20$
D-DFE	11705	42738	196205
I-DFE	14309	53022	236849
IS-DFE	9434	33522	158849
Mimb-Dfe	5713	11023	25705

Table 1: Approximative total computational complexities for the example, for different number of antennas M.

The D- and I-DFE:s require the largest number of computations. The IS-DFE requires slightly less computations and achieves, for the scenarios considered, the best performance. The MIMB-DFE requires the least amount of computations but has a lower performance than the IS-DFE. The MIMB-DFE, however, has a performance comparable to or better than the I-DFE for the scenarios considered. When examining Table 1 we can see that the computational complexities of the D-, I- and IS-DFE:s grow rapidly with an increasing number of antennas. The same effect occurs if the the feedforward filter length, $ns + 1$, is increased. From a complexity point of view the MIMB-DFE could be an interesting alternative when the number of antennas is large and/or the lengths of the feedforward filters are large.

V. Concluding discussion

Different ways of performing joint spatial and temporal equalization have been considered.

Tuning all of the equalizer parameters joinly as in the direct D-DFE and the indirect I-DFE, requires the largest amount of computations. For the scenarios considered they did not however achieve the best performance. The I-DFE had a better performance then the D-DFE. The IS-DFE uses an indirect method and considers only the spatial correlations of the noise plus interference. The number of computations is somewhat reduced and the performance for the the scenarios considered was improved or comparable to the I-DFE. By separating the tuning of the DFE into separate beamformers and later combining the outputs, as in the MIMB-DFE, the computational complexity can be reduced further at the expense of a performance degradation. The MIMB-DFE is primarily interesting, from a complexity point of view, when the number of antennas is large and/or the lengths of the feedforward filters are large. The MIMB-DFE can, better than the MIB-DFE, handle a situation where two (or more) delayed versions of the training sequence is arriving from each direction to the receiver antenna. The performance of the algorithms presented will, of course, depend on the scenarios. This is especially true for the non-general algorithms as, for example, the IS-DFE and the MIMB-DFE. It may also be the case that regularization of the D- and I-DFE does not always work well.

The noise and the interferers used in the simulations were temporally white. One can argue that if the main source of disturbance is co-channel interferers, then it suffices to know from which directions they arrive so that they can be nulled out in the spatial domain. The temporal color of the interferers is then of secondary importance. A situation which the IS-DFE may not handle as well as the D- and the I-DFE:s is when there is a significant amount of temporal colored noise present. The IS-DFE, as presented, has no means for combating temporally colored noise other than treating it as white noise.

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