ROBUST DECISION FEEDBACK EQUALIZERS

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Abstract

Design equations are presented for robust and realizable decision feedback equalizers, for IIR channels with coloured noise. Given a probabilistic measure of model uncertainty, the mean MSE, averaged over the whole class of possible models, is minimized. A robustification parameter, which trades off error propagation against theoretical performance, is also introduced. The resulting design equations define a large class of equalizers, with DFE's and linear equalizers based on nominal models being special cases.

If data sequences $\{d(n)\}$ are transmitted in the presence of intersymbol interference, they have to be reconstructed from the received sequences $\{y(n)\}$. Equalizers compute estimates $\bar{d}(n)$ on a symbol by symbol basis. Their main advantage, compared to the MLSE Viterbi detector, is a low computational complexity. If channels are slowly time–varying, filter coefficients can be adjusted during known training sequences, and held fixed until the next training. For fast time–variations, adaptive structures, for example the one described by the right–hand figure below, have to be used.

The conventional approach to adaptive equalization is to make a decision–directed adjustment of the *filter* coefficients directly. An interesting alternative, studied in our project, is *indirect adaptation*: a model of the *channel* (and possibly also of the noise) is adjusted, often with decisioned data $\bar{d}(n)$ being used as channel model input [1].

One advantage of the indirect approach is that in eg. Rayleigh–fading environments, channel parameters change much more "smoothly" than the optimal values of equalizer parameters do. Thus, it is much easier for an adaptive algorithm to track them. See Lindbom (1992). Furthermore, the number of channel parameters is mostly smaller than the required number of equalizer coefficients. Another advantage is that effects of modelling errors can be minimized *analytically* in an indirect approach, as described below.

In an indirect approach, equations are needed for optimizing realizable equalizers for given channel and noise models. For IIR channels with coloured noise, calculation of *linear recursive equalizers* was described in Ahlén and Sternad (1989). Equations for the more high–performance *Decision Feedback Equalizer* (DFE) were presented in Sternad and Ahlén (1990). While simple to use, these methods have two main limitations:

[1] Direct adaptation corresponds to recursive minimization of $|\hat{d}(n) - \bar{d}(n)|^2$ w.r.t. filter parameters, using LMS or RLS. Indirect methods adjust a channel model, with output $\hat{y}(n)$, to minimize $|y(n) - \hat{y}(n)|^2$. Use of *a priori* information to improve tracking of time–varying channels is described in Lindbom (1992) and in the paper in this conference by Lindbom, on page 122.

- They do not take modelling errors into account. The resulting equalizer performance can be sensitive to such errors, in particular for channels with resonance peaks or deep nulls.

- The DFE in Sternad and Ahlén (1990) was optimized under the assumption that past decisions were correct. Sensitivity to erroneous past decisions was not taken into account. The resulting DFE's sometimes have feedback filters with long impulse responses. Long error propagation events may then occur.

Design equations which take these two problems into account will be presented below. The robustification is based on *stochastic representation of the mismodelling and of decision errors.* We describe the received, discrete–time, complex baseband signal $y(n)$ as

$$y(n) = \left(\frac{B_0(q^{-1})}{A_0(q^{-1})} + \frac{\Delta B(q^{-1})}{A_1(q^{-1})} \right) d(n-k) + w(n) \tag{1}$$

with q^{-1} being the backward shift operator. The transmitted symbols $\{d(n)\}$ are assumed to be zero mean and white, with variance $E|d(n)|^2 = \sigma_d^2$. The noise $w(n)$ is described by

$$w(n) = \left(\frac{M_0(q^{-1})}{N_0(q^{-1})} + \frac{\Delta M(q^{-1})}{N_1(q^{-1})} \right) v(n) \tag{2}$$

where $v(n)$ is zero mean and white, with (uncertain) standard deviation σ_v. In these time–invariant models, B_0/A_0 and M_0/N_0 represent stable and known *nominal models*, while $\Delta B/A_1$ and $\Delta M/N_1$ are members of model error classes. Coefficients of their numerator polynomials, eg. $\Delta B(q^{-1}) = \Delta b_0 + \Delta b_1 q^{-1} + \ldots + \Delta b_{\delta b} q^{-\delta b}$, are seen as (time-independent) stochastic variables, with zero means and known covariance matrices. The stable denominators A_1 and N_1 are fixed. In Sternad and Ahlén (1993), such representations are shown to be suitable for describing a wide range of model error types. They are related to the stochastic embedding approach of Goodwin and Salgado (1989). For example, a FIR channel with white noise is described by

$$y(n) = (B_0(q^{-1}) + \Delta B(q^{-1}))d(n-k) + v(n) \quad .$$

Assume a nominal model $\hat{y}(n) = B_0(q^{-1})d(n-k)$ to be estimated by the least squares method and the order of B_0 to be adequate. Then, elements of the LS covariance matrix can serve directly as estimates of covariances $E(\Delta b_i \Delta b_j^*)$.

Introduce an IIR decision feedback equalizer

$$\hat{d}(n-\ell|n) = \frac{S(q^{-1})}{R(q^{-1})} y(n) - \frac{Q(q^{-1})}{P(q^{-1})} \bar{d}(n-\ell-1) \tag{3}$$

where ℓ is a user–chosen smoothing lag and $\bar{d}(n)$ is decisioned data. The denominator polynomials $R(q^{-1})$ and $P(q^{-1})$ are assumed to be monic and required to be stable. The errors in the decisioned data $\bar{d}(n)$ will be treated as uncertainty and represented by an *additive white noise* $\kappa(n)$, uncorrelated with $d(n-j)$ and $v(n-j)$ for all j [2]

$$\bar{d}(n) = d(n) + \kappa(n) \quad ; \quad E|\kappa(n)|^2 = \eta\sigma_d^2 \quad . \tag{4}$$

[2] This is, of course, a simplification. In reality, the error $\kappa(n)$ is non–stationary since decision errors tend to occur in bursts. There may also exist correlations to past noise samples, in particular to those that caused the error. These nonlinear and time–varying effects are neglected here, to obtain a user–friendly tuning parameter.

The scale factor η is used to trade off error propagation against theoretical performance, with $\eta = 0$ representing a belief in error–free decisioned data. Use of a small positive value of η often gives a lower bit error rate [3]. Now, a single equalizer is to be optimized with respect to the whole model error class. We will minimize an *averaged MSE criterion*

$$J = \bar{E}E|d(n-\ell) - \hat{d}(n-\ell|n)|^2 \tag{5}$$

where E represents expectation over noise and $\bar{E}$ is an expectation over the model error distribution in (1) and (2). This type of criterion has been used in connection to other filtering problems, e.g. by Chung and Bélanger (1976). Note that not only the *range* of uncertainties, but also their *likelihood* is taken into account by (5); common model deviations will have a greater impact on an estimator design than do very rare "worst cases". Compared to the use of a minimax design, the conservativeness is thus reduced.

For an ensemble of systems (1),(2), simple design equations have been derived for minimizing the criterion (5) with respect to the coefficients of (3), assuming (4). (A novel derivation technique, described in Ahlén and Sternad (1991), was used.) For polynomials $P(q^{-1})$, let $P_* \triangleq P^*(q)$. Define polynomials $H(q^{-1}), A(q^{-1})$ and $N(q^{-1})$ as

$$H \triangleq B_0 A_1 N_0 N_1 \quad ; \quad A \triangleq A_0 A_1 \quad ; \quad N \triangleq N_0 N_1 \quad .$$

Define double–sided polynomials $\tilde{B}\tilde{B}_*(q, q^{-1})$ and $\tilde{M}\tilde{M}_*(q, q^{-1})$ by

$$\begin{aligned} \tilde{B}\tilde{B}_* &\triangleq B_0 B_{0*} A_1 A_{1*} + \bar{E}(\Delta B \Delta B_*) A_0 A_{0*} \\ \tilde{M}\tilde{M}_* &\triangleq M_0 M_{0*} N_1 N_{1*} + \bar{E}(\Delta M \Delta M_*) N_0 N_{0*} \quad . \end{aligned} \tag{6}$$

Here, $\bar{E}(\Delta B \Delta B_*)$ and $\bar{E}(\Delta M \Delta M_*)$ are easily computed from the covariance matrices. Finally, define $\rho = \bar{E}(\sigma_v^2)/\sigma_d^2$. Let the scalar r_1 and the stable monic polynomial $\beta(q^{-1})$ be the solution to the *averaged spectral factorization*

$$r_1 \beta \beta_* = N N_* A_0 A_{0*} \bar{E}(\Delta B \Delta B_*) + \eta N N_* \tilde{B}\tilde{B}_* + (1+\eta)\rho A A_* \tilde{M}\tilde{M}_* \quad . \tag{7}$$

Let $\{Q(q^{-1}), S_1(q^{-1}), L_{1*}(q), L_{2*}(q)\}$ be the *unique* solution to the *coupled polynomial equations*

$$\begin{aligned} \beta + q^{-1}(1+\eta)Q &= q^{\ell-k} H S_1 + \beta L_{1*} && (8) \\ -q^{-\ell+k}\eta H_* + q(1+\eta)L_{2*} &= -r_1 \beta_* S_1 + q^{-\ell+k} H_* L_{1*} \quad . && (9) \end{aligned}$$

Then, the equalizer (3), which minimizes (5), is given by Q from (8) and

$$S = S_1 N A \quad ; \quad R = \beta \quad ; \quad P = \beta \quad . \tag{10}$$

Solution of the spectral factorization (7) is straightforward. The coupled equations (8),(9) can be solved in precisely the same way as the corresponding equations (3.3a,b) in Sternad and Ahlén (1990). Convert them to systems of linear equations. Then, a new system of linear equations is created by combining all equations with known left-hand sides. This system is solved for the coefficients of S_1 and L_{1*}. Subsequently, Q is obtained from (8). The degrees are $nS_1 = nL_1 = \ell - k, nQ = nL_2 = \max\{nH, n\beta\} - 1$.

[3] For a related suggestion, using a linear combination of a zero forcing linear equalizer and a zero forcing DFE, see Jenq (1979).

Note that, apart from the nominal model, ρ and η, only the second order moments of the model error distributions need to be known. If not known, A_1, N_1, and the covariance matrices of ΔB and ΔM can still be used as "robustness tuning knobs". In the case of no model uncertainty, we set $\Delta B = \Delta M = 0, A_1 = N_1 = 1$ in (6)–(10). An increase of the covariance matrix of ΔB or ΔM will result in more cautious feedback and feedforward filters, with lower gains and lower, broader, spectral peaks. An increase of η reduces the gain of the feedback filter Q/P. The equations (3)–(10) define a class of robust equalizers, with linear equalizers and DFE's as special cases:

- If $\eta = 0$ (perfect decisions assumed), and with no model uncertainty, the *IIR DFE* discussed in Sternad and Ahlén (1990) is obtained [4]. In this (and only this) case, the solution of a spectral factorization (7) is not required. (We get $\beta = A_0 M_0$.)
- If $\eta \to \infty$ (decisioned data are very unreliable), $|Q(q^{-1})| \to 0$. Then, (7), (9) reduce to the design equations for a *robust linear equalizer* S/R, derived in Sternad and Ahlén (1993). (Divide (7)–(9) by η and set $r \triangleq r_1/\eta$, which is finite.)
- When $\eta \to \infty$ and no model uncertainty is assumed, we obtain the ordinary linear recursive equalizer.

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[4]We then have $\tilde{B}\tilde{B}_* = B_0 B_{0*}$, $\tilde{M}\tilde{M}_* = M_0 M_{0*}$, $r_1 = \rho$, $H = B_0 N_0 \triangleq \tau$ and $\beta = A_0 M_0 \triangleq \gamma$.