Scalability of Bidirectional Vehicle Strings with Measurement Errors

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Abstract
Poor scalability arises in many vehicle platoon problems. Bidirectional strings appear to show some promise for mitigating these problems. In some cases these solutions have the undesirable side effect of non-scalable response to measurement errors. In this paper, we examine this problem and show how information exchange between vehicles may eliminate scalability difficulties due to measurement errors.

Keywords: scalability, string stability, vehicle strings, measurement errors

1. INTRODUCTION

In the field of coordinated systems, formation control is a well studied control objective. In its simplest form a group of N vehicles (e.g. platoon or string) is required to move in one direction and follow a given reference trajectory while the vehicles keep a prescribed distance to neighbouring vehicles.

It is usually desirable to find distributed control solutions, using local measurements only. In this paper bidirectional distributed control of a string is studied, see e.g. Seiler et al. [2004], Barooah et al. [2009].

It is well known that error signals can amplify when travelling through the string resulting in growth of the local error norm with the position in the string. This effect is referred to as 'string instability', e.g. in Darbha and Hedrick [1996], Seiler et al. [2004], 'slinky effect', e.g. in Zhang et al. [1999] or' not scalable', e.g. in Lestas and Vinnicombe [2007].

It was shown in Seiler et al. [2004], Barooah and Hespanha [2005] that linear symmetric bidirectional strings with two integrators in the open loop and constant spacing are always string unstable. Lestas and Vinnicombe [2007] examines a bidirectional string with constant spacing and shows that string stability can be achieved with sufficiently large coupling with the leader position.

In a different approach a symmetric bidirectional string was modelled as a mass-spring-damper system in Eyre et al. [1998]. This idea was extended in Knorn et al. [2013] using the theory of port-Hamiltonian systems (PHS). It was shown that the analysis of stability and string stability of systems in this form is significantly simplified. Also, sufficient conditions to guarantee l_2 string stability of such systems were derived.

However, all results discussed above assume perfect and accurate measurements of all relevant and necessary states. In this paper we will study a system with simple, constant measurement offsets in the sensors measuring the distance towards the neighbouring vehicles. When assuming different measuring errors for the same inter vehicle distance (due to two different sensors used in each of the vehicles) offsets might accumulate. The undesirable effect can be an equilibrium that grows without bound as the string size increases. We show that this problem can be avoided if a simple consensus algorithm is implemented. Note that a similar problem might arise in case different steady state distances are assumed by neighbouring vehicles. For simplicity, however, we will concentrate our discussion on measurement offsets in the reminder of this paper. Similar issues regarding the effects of time varying measurement noise were studied in Bamieh et al. [2012], Hao and Barooah [2012].

The remainder of the paper is organised as follows: The model and the notation are discussed in Section 2. Results from Knorn et al. [2013] on stability and scalability of vehicle platoons are summarised in Section 3. Section 4 studies stability and string stability in case of unknown measurement offsets. The paper closes with three illustrative scenarios in Section 5 and conclutions in Section 6.

2. PLATOON MODEL AND NOTATION

2.1 Notation

We consider a string of N vehicles driving behind one another. The mass of the *ith* vehicle is m_i and its motion can be described by its momentum and position, p_i and q_i respectively, with $i = 1,2,\cdots,N$. Thus,

$$\dot{p}_i = F_i + d_i, \quad \text{and} \quad \dot{q}_i = m_i^{-1} p_i. \tag{1}$$

where F_i is the control force on the vehicle and d_i is the disturbance. The control force F_i will be chosen such that only data from a group of neighbours of the ith vehicle (both preceding and following vehicles) are needed. Hence, no global communication structure is necessary.

We denote the state, steady state and the disturbance column vectors (generally denoted col) by $x(t) = \text{col}(x_1(t),\ldots,x_N(t))$, $x_0 = \text{col}(x_{1_0},\ldots,x_{N_0})$ and $d(t) = \text{col}(d_1(t),\ldots,d_N(t))$. The column vector of ones is denoted by $\underline{1}$ and $\underline{e}_i$ is the ith canonical vector of length N. Similarly we denote the diagonal matrix $A \in$

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$\mathbb{R}^{N\times N}$ with diagonal entries $a_1,\dots a_N$ as $A = \mathrm{diag}(a_1,\dots a_N)$. The L_2 vector norm is denoted by $|x|_2 = |x| = \sqrt{x^{\mathrm{T}}x}$ and the L_2 and L_∞ vector function norms by $\|x(\cdot)\|_2 = \sqrt{\int_0^\infty |x(t)|^2\,\mathrm{d}t}$ and $\|x(\cdot)\|_\infty = \sup_{t\geq 0}|x(t)|$, respectively. The gradient of H is denoted by ∇H. The transpose of a matrix A is denoted A^{T}, the inverse of A by A^{-1} and the inverse of its transpose by $A^{-\mathrm{T}}$.

2.2 Control Objectives

The local control objective for each vehicle is to bring its local error to zero via distributed control based solely on locally available data. The local error is usually defined as a linear combination of position errors (e.g. distances or displacements) towards a limited group of direct predecessors and a limited group of directly following vehicles. The controller for the first vehicle in the string aims to follow a given trajectory q_0 and also minimise the local position error towards a group of following vehicles. In the simplest setting the reference signal is considered to be a ramp with constant velocity v_0, i.e. $q_0 = v_0 t$. Note that only a small, limited group at the beginning of the string have direct access to the reference signal. In our setting, there is no global communications, and therefore neither the reference position, velocity v_0, nor any data of the leading vehicle are available to every other vehicle in the string.

The overall control objective is to achieve "string stability" or "scalability", that is, the norm of the local states of the complete string are bounded, uniformly in the string size N, for nonzero disturbances or initial conditions, Darbha and Hedrick [1996]:

Definition 1. The equilibrium x^* of a distributed system with N agents is l_2 string stable with respect to disturbances $d(t)$, if given any $\epsilon > 0$, there exists $\delta_1(\epsilon) > 0$ and $\delta_2(\epsilon) > 0$ such that

$$|x(0) - x^*| < \delta_1(\epsilon) \quad \text{and} \quad \|d(\cdot)\|_2 < \delta_2(\epsilon) \tag{2}$$

implies

$$\|x(t) - x^*\|_\infty = \sup_{t\geq 0}|x(t) - x^*| < \epsilon \tag{3}$$

for all $N \geq 1$.

Note that in the literature implicitly or explicitly different definitions for string stability are known. For instance it is shown in Seiler et al. [2004], Barooah and Hespanha [2005] that a bidirectional vehicle string with nearest neighbour communication only and tight spacing policy cannot be string stable. The definition used there is that the L_2 norm of $x(t) - x^*$ is bounded independently of N. In contrast, the L_∞ norm is used in this work.

2.3 The Uncontrolled System

In this section we briefly review how the uncontrolled vehicle string (described by (1)) can be written as a port-Hamiltonian system, see e.g. Ortega and Romero [2012]. This will facilitate the design of control algorithms and the proof of stability and string stability later.

Since the explicit control objective is to obtain zero position errors / displacements between the vehicles (and therefore an implicit control objective is for each vehicle to drive with velocity v_0), we choose the local displacements $\Delta_i = q_{i-1} - q_i$ and the momenta $p_i = m_i v_i$ as the system states. Note that $\Delta_1 = q_0 - q_1$ where the position q_0 is the product of the constant velocity reference v_0 and time. Using (1) this yields

$$\dot{\Delta}_i = \dot{q}_{i-1} - \dot{q}_i = m_{i-1}^{-1}p_{i-1} - m_i^{-1}p_i. \tag{4}$$

Thus, with (1) and (4) the system dynamics can be written as

$$\begin{bmatrix}\dot{p}\\ \dot{\Delta}\end{bmatrix} = \begin{bmatrix}0 & S^{\mathrm{T}}\\ -S & 0\end{bmatrix}\nabla H(p,\Delta) + \begin{bmatrix}F\\ 0\end{bmatrix} + \begin{bmatrix}d\\ \underline{e}_1 v_0\end{bmatrix}, \tag{5}$$

where $\Delta, p \in \mathbb{R}^N$ are the displacement and momentum vectors, i.e. $\Delta = \mathrm{col}(\Delta_1,\dots,\Delta_N)$, $p = \mathrm{col}(p_1,\dots,p_N)$, the control force vector is $F = \mathrm{col}(F_1,\dots,F_N)$, the function H is given by $H(p,\Delta) = \frac{1}{2}p^{\mathrm{T}}M^{-1}p$, the matrix $M \in \mathbb{R}^{N\times N}$ is the constant and positive definite inertia matrix $M = \mathrm{diag}(m_1,\dots,m_N)$ and the matrix S has the bidiagonal form

$$S = \begin{bmatrix}1 & 0 & \cdots & \cdots & 0\\ -1 & 1 & \ddots & & \vdots\\ 0 & -1 & \ddots & \ddots & \vdots\\ \vdots & \ddots & \ddots & 1 & 0\\ 0 & \cdots & 0 & -1 & 1\end{bmatrix}. \tag{6}$$

3. CONTROLLER DESIGN FOR VEHICLE STRINGS USING PORT-HAMILTONIAN SYSTEM THEORY

We now give a brief review of key results (see for example Knorn et al. [2013]) using Port-Hamiltonian theory for vehicle platoon results. This is an essential precursor to our later results on the effects of measurement errors in such systems.

3.1 Local Control

The local control is motivated by results from mechanical engineering. When the control actions between the vehicles is chosen such that they can be understood as virtual springs and dampers between the vehicles the overall system can be written as a port-Hamiltonian system.

The control forces consist of the "spring force" F_i^{s}, that depends linearly on the position errors Δ_i, the "damper force" F_i^{r}, that depends linearly on the velocity errors between two neighbouring vehicles, and the "drag force" F_i^{d} describing a virtual friction of vehicle i towards the ground:

$$\begin{aligned}F_i =& F_i^{\mathrm{s}} - F_{i+1}^{\mathrm{s}} - F_i^{\mathrm{d}} + F_i^{\mathrm{r}} - F_{i+1}^{\mathrm{r}}\\ =& c_i^{-1}\Delta_i - c_{i+1}^{-1}\Delta_{i+1} - b_i m_i^{-1}p_i\\ &+ R_i(m_{i-1}^{-1}p_{i-1} - m_i^{-1}p_i) - R_{i+1}(m_i^{-1}p_i - m_{i+1}^{-1}p_{i+1})\\ &\forall i = 1,\cdots,N-1,\end{aligned} \tag{7}$$

$$\begin{aligned}F_N =& F_N^{\mathrm{s}} + F_N^{\mathrm{r}} - F_N^{\mathrm{d}}\\ =& c_N^{-1}\Delta_N + R_N(m_{N-1}^{-1}p_{N-1} - m_N^{-1}p_N) - b_N m_N^{-1}p_N\end{aligned} \tag{8}$$

such that we can write

$$F = -(B+R)M^{-1}p + \underline{e}_1 R_1 v_0 + S^{\mathrm{T}}C^{-1}\Delta \tag{9}$$

with the constant matrices,

$$R = \begin{bmatrix}R_1+R_2 & -R_2 & 0 & \cdots & 0\\ -R_2 & R_2+R_3 & -R_3 & \ddots & \vdots\\ 0 & -R_3 & \ddots & \ddots & 0\\ \vdots & \ddots & \ddots & R_{N-1}+R_N & -R_N\\ 0 & \cdots & 0 & -R_N & R_N\end{bmatrix}, \tag{10}$$

$B = \mathrm{diag}(b_1,\dots,b_N)$ and $C = \mathrm{diag}(c_1,\dots,c_N)$, where the entries of matrices B, R and C are design parameters of the controller and $0 < R_i, b_i, c_i < \infty$ for all i.

It has been shown in [Knorn et al., 2013, Lemma 1] that given $d = 0$ the equilibrium $(M\underline{1}v_0, CS^{-\mathrm{T}}B\underline{1}v_0)$ of system (5) in closed loop with the controller (7)-(8) is asymptotically stable. (The proof can be found in Appendix A at the end of this paper.) However, this implies that the displacements Δ, for vehicles towards the front of the string, grow with the string size N since

$$S^{-\mathrm{T}} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 \\ 0 & \dots & 0 & 1 \end{bmatrix} \tag{11}$$

and hence $\Delta_i^* = c_i \sum_{k=i}^{N} b_k v_0$. Thus, the system converges to a undesirable equilibrium $\Delta^* \neq 0$ if any $b_i > 0$. This problem could be avoided by choosing $b_i = 0$, however, in doing so, l_2 string stability with respect to disturbances cannot be guaranteed. In the next subsection suitable integral action will be added to the local control to ensure l_2 string stability with respect to disturbances of the desired equilibrium.

3.2 Integral Action

To avoid undesirable growth of the equilibrium states, integral action is added to the local control algorithm. Thus, the complete control is described by

$$F = -(B+R)M^{-1}p + \underline{e}_1 R_1 v_0 + S^{\mathrm{T}}C^{-1}\Delta + F_{\mathrm{IA}} \tag{12}$$

$$F_{\mathrm{IA}} = MKS^{\mathrm{T}}C^{-1}\Delta - (B+R)Kz_3 \tag{13}$$

$$\dot{z}_3 = -S^{\mathrm{T}}C^{-1}\Delta. \tag{14}$$

where $K \in \mathbb{R}^{N\times N}$ is a diagonal positive matrix $K = \mathrm{diag}(k_1, \dots, k_N)$. Assume further that the disturbance d include a constant component d_{c} and a time-varying disturbance $d_{\mathrm{d}}(t)$ such that $d = d_{\mathrm{c}} + d_{\mathrm{d}}(t)$ and there exists a constant $D < \infty$ satisfying $||d_{\mathrm{d}}(\cdot)||_2 \leq D$. Then, it can be shown, [Knorn et al., 2013, Lemma 3], that

(1) for $d_{\mathrm{d}}(t) = 0$ the desired equilibrium

$$(p^*, \Delta^*, z_3^*) = (M\underline{1}v_0, 0, \alpha) \tag{15}$$

with $\alpha = K^{-1}(B+R)^{-1}(d_{\mathrm{c}} - B\underline{1}v_0)$ is globally asymptotically stable (despite the presence of constant unknown disturbances d_{c}), and

(2) the equilibrium (15) is l_2 string stable for disturbances $||d_{\mathrm{d}}(\cdot)||_2$ satisfying $||d_{\mathrm{d}}(\cdot)||_2 < \infty$.

An outline of a proof can be found in Appendix B at the end of this paper.

4. MEASUREMENTS AND MEASUREMENT OFFSETS

4.1 Background on Measurement Offsets

The previous section described how a vehicle string system with exact measurements can be controlled using local control and integral action. Using integral action yields an l_2 string stable system for accurate measurements.

However, even small measurement offsets at each vehicle can accumulate and lead to an undesired equilibrium. As will be shown below in Section 4.2, l_2 string stability of the desired equilibrium in the presence of measurement offsets and the control structure above cannot be guaranteed.

Assume that the distances between the vehicles Δ are measured locally by each vehicle using radar sensors. The distance between vehicle i and its predecessor, Δ_i is measured by both vehicle i and the predecessor, that is vehicle $i-1$. Assume further that both sensors operate independently of each other. Thus, it is possible for both sensors to have a constant distinct offset. Hence the overall measurement of the front distance, $\Delta_{\mathrm{m,f},i}$, and the back distance , $\Delta_{\mathrm{m,b},i}$, are described by

$$\Delta_{\mathrm{m,f},i} = \Delta_i + \hat{\Delta}_{\mathrm{f},i} \quad \text{and} \quad \Delta_{\mathrm{m,b},i} = \Delta_i + \hat{\Delta}_{\mathrm{b},i} \tag{16}$$

with $|\hat{\Delta}_{\mathrm{f},i}| \leq \delta$ and $|\hat{\Delta}_{\mathrm{b},i}| \leq \delta$ for all i where $\delta < \infty$ is the upper bound on the measurement offsets. The overall measurement vectors thus are

$$\Delta_{\mathrm{m,f}} = \Delta + \hat{\Delta}_{\mathrm{f}} \quad \text{and} \quad \Delta_{\mathrm{m,b}} = \Delta + \hat{\Delta}_{\mathrm{b}}. \tag{17}$$

where $\hat{\Delta}_{\mathrm{f}} = \left[\hat{\Delta}_{\mathrm{f},1}\ \hat{\Delta}_{\mathrm{f},2}\ \dots\ \hat{\Delta}_{\mathrm{f},N}\right]^{\mathrm{T}}$ and $\hat{\Delta}_{\mathrm{b}} = \left[0\ \hat{\Delta}_{\mathrm{b},2}\ \dots\ \hat{\Delta}_{\mathrm{b},N}\right]^{\mathrm{T}}$. Note that apart from the lead vehicle, no agent measures the distance between the first vehicle and the reference trajectory, Δ_1. Thus, $\hat{\Delta}_{\mathrm{b},1} = 0$.

An alternative measurement and communication topology (i.e. "distance measurement consensus"), which we consider later in Section 4.3, allows basic communication between neighbouring vehicles. If both vehicles then choose the algebraic mean of both values instead of their local measurement, the local measurement reduces to $\Delta_{\mathrm{m},i} = \Delta_i + \tilde{\Delta}_i$ with $|\tilde{\Delta}_i| \leq \delta$ for all i. Thus, the overall measurement vector is

$$\Delta_{\mathrm{m}} = \Delta + \tilde{\Delta}. \tag{18}$$

4.2 Effect of Measurement Offsets

Results in Subsection 3.2 revealed how string stability / scalability of the system can be guaranteed when using appropriate integral action control laws. However, if exact measurements are not available (but unknown, constant measurement offsets are present), the system diverges to a different equilibrium point.

Lemma 2. Consider a string of N vehicles with local control including integral action control given in (12)-(14) as discussed in Section 3. Assume the first vehicle is following a reference trajectory with velocity v_0. If the system is subject to unknown constant disturbances d_{c} and unknown measurement offset as in (17), then the stable equilibrium of the system is

$$\left(p^*, \Delta^*, z_3^*\right) = \left(M\underline{1}v_0, -CS^{-\mathrm{T}}\hat{\Delta}, \alpha\right) \tag{19}$$

with

$$\hat{\Delta} = C^{-1}\hat{\Delta}_{\mathrm{f}} + (S^{\mathrm{T}} - I)C^{-1}\hat{\Delta}_{\mathrm{b}} \tag{20}$$

and

$$\alpha = K^{-1}(B+R)^{-1}(d_{\mathrm{c}} - B\underline{1}v_0). \tag{21}$$

(The proof can be found in Appendix C.) Note that the constant disturbances d_{c} do not influence the equilibrium of Δ. However, Δ^* depends on the constant measurement offsets. Namely,

$$\Delta^* = -CS^{-\mathrm{T}}\hat{\Delta} = -CS^{-\mathrm{T}}\left(C^{-1}\hat{\Delta}_{\mathrm{f}} + (S^{\mathrm{T}} - I)C^{-1}\hat{\Delta}_{\mathrm{b}}\right) \tag{22}$$

implies

$$\Delta_i^* = \sum_{k=i+1}^{N} \frac{c_i}{c_k}\hat{\Delta}_{\mathrm{b},k} - \sum_{k=i}^{N} \frac{c_i}{c_k}\hat{\Delta}_{\mathrm{f},k}. \tag{23}$$

Thus, even though the system is stable (and by definition also l_2 string stable) the resulting equilibrium is highly undesirable as it involves severe safety risks: Note that, the measurement offsets might accumulate at the beginning of the string. The distance between the first and the second vehicle in the string can grow without bound. If the forward measurement offset on all vehicles is consistently *greater* than the backwards measurement offset, the distance between the first two vehicles gets smaller when the string size increases. Thus, for an increasing string length the cars at the beginning of the string crash. In case the forward measurement offset on all vehicles is consistently *smaller* than the backwards measurement offset, the displacement errors at the beginning of the string grow without bound when N increases. Although this case does not result in pile up crashes, this effect is also undesirable since it corresponds to string break up.

In some cases, one might assume that *on average* the sum of forward and backwards measurement offsets will be close to zero even for a long string. However, even if this is true for the expected value of the steady state error, the variance of the steady state error will still grow without bound as N increases. To see this, assume all forward and backwards measurement offsets are independent, have an expected value of 0 and a variance of $\mathrm{Var}\left(\hat{\Delta}_{\mathrm{b},i}\right) = \mathrm{Var}\left(\hat{\Delta}_{\mathrm{f},i}\right) = \sigma^2 < \infty$ for all i. Further assume that the measurement offsets are uncorrelated to each other. Then

$$\begin{aligned}\mathrm{Var}\left(\Delta_i^*\right) =& \mathrm{Var}\left(\sum_{k=i+1}^{N} \frac{c_i}{c_k}\hat{\Delta}_{\mathrm{b},k}\right) + \mathrm{Var}\left(\sum_{k=i}^{N} \frac{c_i}{c_k}\hat{\Delta}_{\mathrm{f},k}\right)\\ =& \sum_{k=i+1}^{N}\left(\frac{c_i}{c_k}\right)^2 \mathrm{Var}\left(\hat{\Delta}_{\mathrm{b},k}\right) + \sum_{k=i}^{N}\left(\frac{c_i}{c_k}\right)^2 \mathrm{Var}\left(\hat{\Delta}_{\mathrm{f},k}\right)\\ =& \sigma^2\left(2\sum_{k=i+1}^{N}\left(\frac{c_i}{c_k}\right)^2 + 1\right).\end{aligned} \tag{24}$$

Note that for $i = 1$ and $c_k = c$ for all k, $\mathrm{Var}\left(\Delta_1^*\right)$ grows linearly with N. This effect could be reduced by choosing decreasingly stiff springs between the vehicles towards the end of the string. However, to avoid variance growth with string length, the compliance coefficients have to decrease drastically with the position within the string. This is clearly undesirable from a practical point of view and would lead to other complications such as much slower settling times towards the end of the string. Also, in this setting global knowledge is required since every agent (e.g. vehicle) needs to know its position within the string.

Unfortunately neither the correct distance between neighboring vehicles nor the measurement offset can be observed through $y = \Delta + \hat{\Delta}$. To see that consider the system

$$\begin{pmatrix}\dot{p}\\ \dot{\Delta}\\ \dot{\hat{\Delta}}\end{pmatrix} = \begin{bmatrix}0 & 0 & 0\\ -S\,M^{-1} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{pmatrix}p\\ \Delta\\ \hat{\Delta}\end{pmatrix} + \begin{bmatrix}I\\0\\0\end{bmatrix}u, \quad y = \begin{bmatrix}0 & I & I\end{bmatrix}\begin{pmatrix}p\\ \Delta\\ \hat{\Delta}\end{pmatrix}. \tag{25}$$

The observability matrix, $\mathcal{O} = \begin{bmatrix}0 \; I \; I; -S\,M^{-1} \; 0 \; 0; 0 \; 0 \; 0\end{bmatrix}$, clearly does not have full rank. While building an observer might help to filter measurement noise it cannot avoid convergence to the undesirable equilibrium due to accumulated measurement offsets.

4.3 Distance measurement consensus

In the following lemma we will show that l_2 string stability of a bounded equilibrium can be guaranteed if all vehicles reach “distance measurement consensus” with their direct neighbours.

Lemma 3. Consider a string of N vehicles with local control including integral action given in equations (12)-(14) as discussed in Section 3. Assume the first vehicle is following a reference trajectory with velocity v_0. If the system is subject to unknown constant disturbances d_c and unknown measurement of the form described in (18) then the equilibrium of the system

$$\left(p^*, \Delta^*, z_3^*\right) = \left(M\underline{1}v_0, -\tilde{\Delta}, \alpha\right) \tag{26}$$

with

$$\alpha = K^{-1}(B+R)^{-1}\left(d_\mathrm{c} - B\underline{1}v_0\right). \tag{27}$$

is asymptotically stable. Further, the equilibrium is l_2 string stable for any additional time-varying disturbance $d_\mathrm{d}(t)$ satisfying $\|d_\mathrm{d}(\cdot)\|_2 < \infty$.

The proof can be found in Appendix D.

Hence, l_2 string stability of a bounded equilibrium can be guaranteed by establishing a simple consensus algorithm between neighbouring vehicles. Since the measurement offsets after agreeing on the arithmetic mean for both vehicles is equivalent, it does not accumulate towards the beginning of the string. The disadvantage here is, however, that the boundedness of the equilibrium crucially depends on inter vehicle communication.

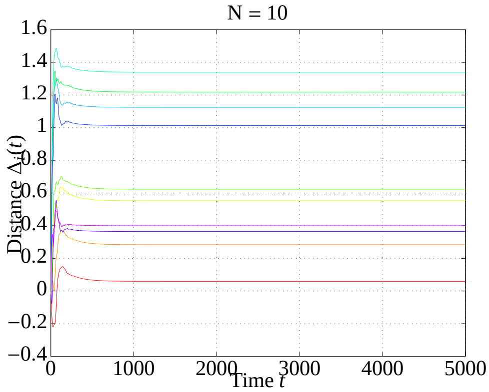

Figure 1. Scenario 1: Δ_i for $i = 1$ (red), 2 (orange), $\cdots$, 10 (purple)

5. EXAMPLE

Two homogeneous bidirectional vehicle strings have been simulated. The first is of length $N = 10$ while the second contains $N = 100$ vehicles. In both cases the first vehicle is required to follow a given trajectory with $q_0 = v_0 t$ and all vehicles start with initial values being zero both for the velocity v and the displacements Δ. The measurement of Δ is subject to randomised (values vary between 0 and 1) measurement offsets both in the forward and the backwards measurements. Two random vectors of length $N = 100$ have been generated to simulate the measurement offsets. For the shorter platoon, only the first 10 entries of the vectors are used.

In the first scenario the measurement offset vectors both are uniformly distributed on [0,1]. Thus, on average the difference between the forward and the backwards measurement offset is zero. While it seems in Fig. 1 that measurement offsets are accumulating in steady state, close examination of Fig. 2 reveals that this is not the case as the string length increases. However, the span of the steady state displacements grows from approximately 0 – 2m for $N = 10$ to −3 – 1.5m for $N = 100$. This could have been expected as the variance in this case grows with the string length N.

In Scenario 2 an additional offset of 0.1m is added to each forward measurement error. As expected in this case (with unbalanced forward and backward measurement offsets) the steady state deviations of delta at the beginning of the string increase with the string length N, Fig. 3.

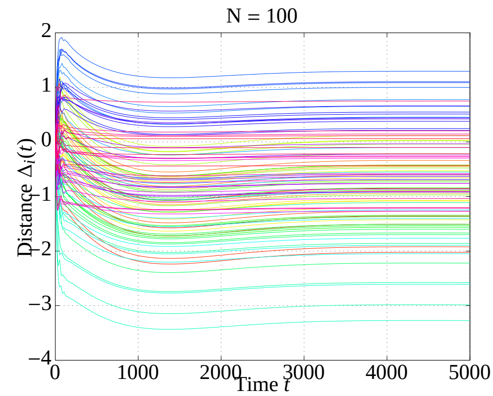

Figure 2. Scenario 1: Δ_i for $i = 1$ (red), 2 (orange), $\cdots$, 100 (purple)

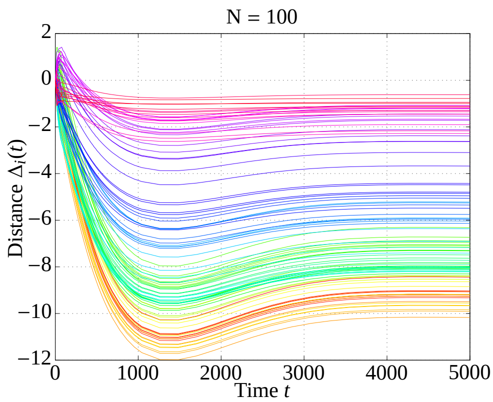

Figure 3. Scenario 2: Δ_i for $i = 1$ (red), 2 (orange), $\cdots$, 100 (purple)

In the third scenario it is assumed that neighbouring vehicles communicate and exchange their deviation measurement with each other. Then the average value of both measurements is used in both vehicles. The effect of this simple additional algorithm can be seen in Fig. 4. In this case, the measurement offsets no longer accumulate at the beginning of the string, and in addition, the variance of the steady state deviations does not increase. This can easily be explained realising that with this algorithm the deviation between both vehicles will be the algebraic mean of two bounded uniformly distributed values. Thus, the resulting deviation in steady state will also be bounded independently of the string length N.

6. CONCLUSIONS

This paper studies the effect of unknown, constant measurement offsets on scalability of bidirectional vehicle platoons. It is shown that under some assumptions measurement offsets might accumulate at the beginning of the string. This might lead to undesirable equilibrium state as the distances between the vehicles can grow without bound even though the measurement errors are assumed to be individually bounded.

It is shown that in this case it is sufficient to implement a simplistic consensus algorithm to guarantee a bounded equilibrium. In case neighbouring vehicles agree on a measurement (instead of each vehicle using their individual measurement), offsets do not affect the behaviour of other vehicles in the string. Thus, measurement offsets do not accumulate at any part of

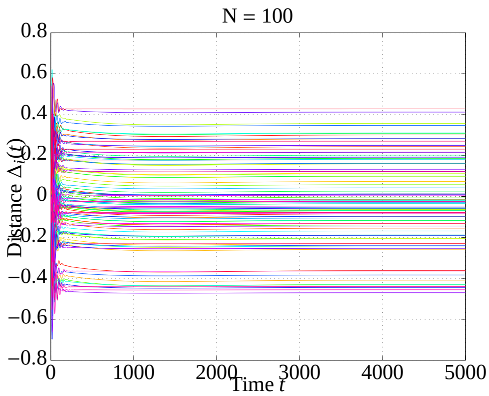

Figure 4. Scenario 3: Δ_i for $i = 1$ (red), 2 (orange), $\cdots$, 100 (purple)

the string. Hence, scalability of the system can be guaranteed despite constant unknown measurement offsets.

In future work, we intent to extend the presented results by investigating the effect of non static measurement error, communication loss and delay between vehicles.

Appendix A. PROOF FOR STABILITY UNDER LOCAL CONTROL

From (9) and (5) the dynamic equations for the closed loop have the form

$$\dot{p} = -(R+B)M^{-1}(p - M\underline{1}v_0) + S^{\mathrm{T}}C^{-1}(\Delta - CS^{-\mathrm{T}}B\underline{1}v_0), \tag{A.1}$$

$$\dot{\Delta} = -S\,M^{-1}(p - M\underline{1}v_0). \tag{A.2}$$

Thus the closed loop has the port Hamiltonian form

$$\begin{bmatrix} \dot{p} \\ \dot{\Delta} \end{bmatrix} = \begin{bmatrix} -(B+R) & S^{\mathrm{T}} \\ -S & 0 \end{bmatrix} \nabla H_{\mathrm{cl}}(p,\Delta), \tag{A.3}$$

with the closed-loop Hamiltonian function

$$\begin{aligned} H_{\mathrm{cl}}(p,\Delta) =& \frac{1}{2}(p - M\underline{1}v_0)^{\mathrm{T}} M^{-1}(p - M\underline{1}v_0) \\ &+ \frac{1}{2}\left(\Delta - CS^{-\mathrm{T}}B\underline{1}v_0\right)^{\mathrm{T}} C^{-1}\left(\Delta - CS^{-\mathrm{T}}B\underline{1}v_0\right). \end{aligned} \tag{A.4}$$

Using $H_{\mathrm{cl}}(p,\Delta)$ as Lyapunov function, and computing the time derivative of $H_{\mathrm{cl}}(p,\Delta)$ yields

$$\dot{H}_{\mathrm{cl}}(p,\Delta) = \nabla^{\mathrm{T}} H_{\mathrm{cl}} \begin{bmatrix} -(B+R) & S^{\mathrm{T}} \\ -S & 0 \end{bmatrix} \nabla H_{\mathrm{cl}} \leq 0 \tag{A.5}$$

since $(B+R) = (B+R)^{\mathrm{T}} > 0$. The biggest invariant set included in $\mathcal{S} = \left\{(p,\Delta) | \dot{H}_{\mathrm{cl}}(p,\Delta) = 0\right\}$ is $(p^*,\Delta^*) = (M\underline{1}v_0, CS^{-\mathrm{T}}B\underline{1}v_0)$. Thus, by LaSalle's Invariance Principle (e.g. [Khalil, 2001, Theorem 4.4]) it can be shown that the system is asymptotically stable and the equilibrium reached is (p^*,Δ^*).

Appendix B. PROOF OUTLINE FOR STABILITY AND L_2 STRING STABILITY WITH INTEGRAL ACTION

B.1 Asymptotic Stability

To show that global asymptotic stability first define the following change of coordinates

$$z_1 = p - M\underline{1}v_0 + MK(z_3 - \alpha), \tag{B.1}$$

$$z_2 = \Delta. \tag{B.2}$$

Combining (A.1), (13) and (14) and using (A.2) yields

$$\dot{z}_1 = -(B+R)M^{-1}z_1 + S^{\mathrm{T}}C^{-1}z_2,$$
$$\dot{z}_2 = -SM^{-1}z_1 + SK(z_3-\alpha).$$

Thus, the closed loop dynamics have the port Hamiltonian form

$$\begin{bmatrix}\dot{z}_1\\\dot{z}_2\\\dot{z}_3\end{bmatrix} = \begin{bmatrix}-(B+R) & S^{\mathrm{T}} & 0\\ -S & 0 & S\\ 0 & -S^{\mathrm{T}} & 0\end{bmatrix}\nabla H_{\mathrm{z}}(z) \tag{B.3}$$

with the Hamiltonian function

$$H_{\mathrm{z}}(z) = \frac{1}{2}z_1^{\mathrm{T}}M^{-1}z_1 + \frac{1}{2}z_2^{\mathrm{T}}C^{-1}z_2 + \frac{1}{2}(z_3-\alpha)^{\mathrm{T}}K(z_3-\alpha). \tag{B.4}$$

Using $H_{\mathrm{z}}(z)$ as Lyapunov function, and computing the time derivative of $H_{\mathrm{z}}(z)$ yields

$$\dot{H}_{\mathrm{z}}(z) = -\nabla^{\mathrm{T}}H_{\mathrm{z}_1}(z)(B+R)\nabla H_{\mathrm{z}_1}(z) \leq 0 \tag{B.5}$$

since $(B+R)=(B+R)^{\mathrm{T}}>0$. The biggest invariant set included in $\mathcal{S}=\left\{z|\dot{H}_{\mathrm{z}}(z)=0\right\}$ is $\left(z_1^*,z_2^*,z_3^*\right)=(0,0,\alpha)$. Thus, by LaSalle's Invariance Principle (see [Khalil, 2001, Theorem 4.4]) it can be shown that the equilibrium $\left(z_1^*,z_2^*,z_3^*\right)$ is asymptotically stable. This implies that the equilibrium in the original coordinates is $\left(p^*,\Delta^*,z_3^*\right)=(M\underline{1}v_0,0,\alpha)$.

B.2 l_2 String Stability

To show that the system is l_2 string stable, choose the Hamiltonian function (B.4). Thus, in presence of dynamical disturbances $d_{\mathrm{d}}(t)$ the closed loop dynamics have the port Hamiltonian form

$$\begin{bmatrix}\dot{z}_1\\\dot{z}_2\\\dot{z}_3\end{bmatrix} = \begin{bmatrix}-(B+R) & S^{\mathrm{T}} & 0\\ -S & 0 & S\\ 0 & -S^{\mathrm{T}} & 0\end{bmatrix}\nabla H_{\mathrm{z}}(z) + \begin{pmatrix}d_{\mathrm{d}}(t)\\0\\0\end{pmatrix}. \tag{B.6}$$

Taking the derivative of H_{z} it can be shown that $\dot{H}_{\mathrm{z}}(z) \leq \frac{1}{2\lambda_{\min}(B+R)}|d_{\mathrm{d}}(t)|^2$. Hence, it follows that (details can be found in Knorn et al. [2013]) $H_{\mathrm{z}}(z(t)) \leq (\min_i(m_i))^{-1}|z_1(0)|^2 + (\min_i(c_i))^{-1}|z_2(0)|^2 + \max_i k_i|z_3-\alpha|^2 + (2\min_i(b_i))^{-1}\|d_{\mathrm{d}}(\cdot)\|^2$. Since the mass m_i, the compliance c_i, the drag coefficient b_i and the integral action control parameter k_i for each vehicle are positive the norm of the states z is bounded for all N if $|z(0)|$ and $\|d(\cdot)\|_2$ do not increase with N. Thus, the system is l_2 string stable.

Appendix C. PROOF OF LEMMA 2

As the measurement of Δ is subject to unknown measurement offset, equations (12)-(14) change to

$$F = -(B+R)M^{-1}p + \underline{e}_1R_1v_0 + S^{\mathrm{T}}C^{-1}\left(\Delta + CS^{-\mathrm{T}}\hat{\Delta}\right) + F_{\mathrm{IA}}$$
$$F_{\mathrm{IA}} = MKS^{\mathrm{T}}C^{-1}\left(\Delta + CS^{-\mathrm{T}}\hat{\Delta}\right) - (B+R)Kz_3$$
$$\dot{z}_3 = -S^{\mathrm{T}}C^{-1}\left(\Delta + CS^{-\mathrm{T}}\hat{\Delta}\right).$$

Choose the following change of coordinates $z_1 = p - M\underline{1}v_0 + MK(z_3-\alpha)$ and $z_2 = \Delta + CS^{-\mathrm{T}}\hat{\Delta}$. Using the fact that for the undisturbed system $\dot{p} = F$ and with (A.2), it follows that, $\dot{z}_1$ yields

$$\begin{aligned}\dot{z}_1 &= -(B+R)M^{-1}(p - M\underline{1}v_0 + MK(z_3-\alpha))\\ &\quad + S^{\mathrm{T}}C^{-1}\left(\Delta + CS^{-\mathrm{T}}\hat{\Delta}\right)\\ &= -(B+R)M^{-1}z_1 + S^{\mathrm{T}}C^{-1}z_2 \end{aligned} \tag{C.1}$$

$$\begin{aligned}\dot{z}_2 &= -SM^{-1}(p - M\underline{1}v_0 + MK(z_3-\alpha)) + SK(z_3-\alpha)\\ &= -SM^{-1}z_1 + SK(z_3-\alpha)\end{aligned} \tag{C.2}$$

Thus, the closed loop dynamics have the port Hamiltonian form (B.3) - (B.4). Using $H_{\mathrm{z}}(z)$ as Lyapunov function, and computing its time derivative yields (B.5) since $(B+R)>0$. The biggest invariant set included in $\mathcal{S}=\left\{z|\dot{H}_{\mathrm{z}}(z)=0\right\}$ is $\left(z_1^*,z_2^*,z_3^*\right)=(0,0,\alpha)$. Thus, by LaSalle's Invariance Principle (see [Khalil, 2001, Theorem 4.4]) it can be shown that the equilibrium $\left(z_1^*,z_2^*,z_3^*\right)$ (and in original coordinates is $\left(p^*,\Delta^*,z_3^*\right)=\left(M\underline{1}v_0,-CS^{-\mathrm{T}}\hat{\Delta},\alpha\right)$) is asymptotically stable.

Appendix D. PROOF OF LEMMA 3

As the measurement of Δ is subject to unknown measurement offset, equations (12)-(14) change to

$$F = -(B+R)M^{-1}p + \underline{e}_1R_1v_0 + S^{\mathrm{T}}C^{-1}(\Delta+\tilde{\Delta}) + F_{\mathrm{IA}} \tag{D.1}$$
$$F_{\mathrm{IA}} = MKS^{\mathrm{T}}C^{-1}\left(\Delta+\tilde{\Delta}\right) - (B+R)Kz_3 \tag{D.2}$$
$$\dot{z}_3 = -S^{\mathrm{T}}C^{-1}\left(\Delta+\tilde{\Delta}\right). \tag{D.3}$$

After the following change of coordinates $z_1 = p - M\underline{1}v_0 + MK(z_3-\alpha)$ and $z_2 = \Delta+\tilde{\Delta}$, very similar steps as in Section C can be followed to show that the equilibrium $\left(z_1^*,z_2^*,z_3^*\right)=(0,0,\alpha)$ is asymptotically stable. This implies that the equilibrium in the original coordinates is $\left(p^*,\Delta^*,z_3^*\right)=\left(M\underline{1}v_0,-\tilde{\Delta},\alpha\right)$. Note that all measurement errors are bound by some δ. Thus, although the system converges to an undesired equilibrium, the equilibrium states do not diverge. Further, similar steps as above can be followed to show that the equilibrium is l_2 string stable.

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