

# Self Focusing of 2D Arrays for SHM of Plate-Like Structures Using Time Reversal Operator

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## ABSTRACT

Lamb waves have shown a great potential in structural health monitoring (SHM) of thin plates that are frequently used in engineering structures. Dense transducer networks or active ultrasonic arrays can be employed to generate and receive Lamb waves. Active arrays, however, enable beam steering and can work as spatial filters enabling assessment of large plate areas from a single fixed position.

In this paper a novel method for selective focusing of Lamb waves is presented. The algorithm is an extension of the DORT method (French acronym for decomposition of time-reversal operator) where the continuous wavelet transform (CWT) is used for the time-frequency representation (TFR) of nonstationary signals instead of the discrete Fourier transform.

An application of the proposed method to self focusing of Lamb waves in an aluminum plate is demonstrated both for a linear and a 2D star-shaped array. The 2D star-like array was designed to eliminate the effect of ambiguous mirrored images encountered for linear arrays. The results obtained with both arrays are presented and compared in the paper. It is shown that the decomposition of the time reversal operator obtained with the proposed method enabled separating point like scatters in the aluminum plate and allowed to focus Lamb waves at a desired point.

## INTRODUCTION

Flat or slightly curved plates can be found in numerous engineered structures where high safety standards are required. Guided waves (GW) offer considerable advantages in monitoring (SHM) applications, where large plate-like areas of the involved structure have to be assessed. GWs that propagate in thin plates, known as Lamb waves, are multimodal and dispersive. Structural discontinuity (e.g., a damage) present in the structure scatters the incident GWs in all directions and additional modes can be produced due to the mode conversion phenomena. Therefore, damage detection and localization is a complex task and advanced signal processing has to be applied to characterize the damage [1].

Some of the difficulties encountered in SHM of plate-like structures resulting from the nature of GWs could be avoided by using the time-reversal concept. For instance, time-reversal mirrors (TRM), a powerful tool introduced for ultrasonic waves [2] [3], enabled improvement of signal to noise ratio [4] and reduction of the effect of dispersion [5][6]. The iterative TRM is a method to obtain a selective focusing on the strongest reflector [7]. However, in the SHM applications focusing on targets with lower reflexivity is also desired. In order to detect and focus the wave on the multiple scatterers DORT method has been introduced [8]. DORT is a signal processing technique that is capable of estimating the time delays required to selective focus waves on a target, based on the received data. The method has been successfully applied in the NDT applications [9] as well as for Lamb wave characterization [10]. In this paper, an extension of the DORT technique that consists in applying the continuous wavelet transform (CWT) to the eigenvectors, is presented. The improved method has been applied to Lamb wave snapshots that normally take the form of nonstationary signals. Using the CWT enabled obtaining time-frequency representation (TFR) of the signals that makes possible both time and frequency localization of the target responses. Based on the modified method a damage detection algorithm employing a 2D star-shaped array was developed. The array consists of a transmission sub-array which can be self-focused on a target and a reception sub-array used to capture damage-reflected echoes.

## THEORETICAL BACKGROUND

In this section a brief review of the DORT technique followed by a description of the proposed extension of the method and its application to damage detection.

### DORT algorithm

DORT is a method of signal analysis capable of detecting and selective focusing elastic waves on point-like scatters with the use of transducer arrays. It was shown that the time reversal operator (TRO), defined as  $K^*K$ , where  $K$  denotes the array transfer function and  $*$  is the complex conjugate, can be diagonalized. Each of the eigenvectors corresponding to the significant eigenvalues of  $K^*K$  includes both phase and amplitude information about the signals that should be applied to the array elements to focus ultrasonic wave on a particular scatterer. Therefore if

$$V_m = [A_1 e^{i\varphi_1}, A_2 e^{i\varphi_2}, \dots, A_n e^{i\varphi_n}]^T \quad (1)$$

is the  $m$ -th eigenvector of the TRO in a monochromatic case the signal that should be applied to an array in order to focus the array's beam on the  $m$ -th scatterer can be calculated using the following equation:

$$s_p(t) = A_p \cos(\omega t - \varphi_p) \quad (2)$$

where  $p$  is the transducer number in the array [11].

## DORT-CWT algorithm

The DORT method enables selective focusing of ultrasonic waves on scatterers in different media. The number of scatterers that can be resolved with the algorithm depends on the size of the TRO matrix  $K^*K$ , which is determined by the number of transducers.

Since the snapshots produced by the propagating Lamb waves are nonstationary due to the dispersion phenomenon, in the proposed algorithm CWT is used to obtain the TFR of the signals. In the algorithm implementation presented here the complex Morlet wavelet was used as the mother wavelet. It enabled obtaining signals decomposed in the complex domain with a very good time and frequency localization. Next, the time-reversal operator,  $K(nT_s)^*K(nT_s)$ , was calculated for the successive time samples at a chosen frequency. In the final step of the DORT-CWT, likewise in the classical DORT, the most significant eigenvalues were selected and the backpropagation was performed. In the proposed method distribution of eigenvalues vs. time samples,  $V_m(nT_s)$ , is obtained. The backpropagation step can be performed, like in the classical approach, with the use of eq. (2). Although each of the eigenvectors calculated for the successive time instants could be backpropagated, only the vectors corresponding to the significant eigenvalues should be taken into account. Even if a threshold value is applied to separate the relevant eigenvalues, still a large number of eigenvectors that should be backpropagated remains. Thorough analysis of the eigenvalues' distributions, performed on our simulated and experimental data revealed that the number of local maxima of the significant eigenvalues is equal to the number of damages existing in the plate. It will be shown below that backpropagation of the vectors corresponding to those maxima enables wave focusing on the damages.

## Numerical backpropagation

The last step of the both DORT and DORT-CWT algorithms is backpropagation of the most significant eigenvectors, which can be performed physically or numerically. In this paper, calculations of backpropagated-fields were based on eq. (2), which defines the relation between the eigenvectors and signals that should be applied to the actuators to focus the wave on a particular target. If the backpropagated field is presented as a bitmap the pixel value  $S_{i,j}$  can be calculated using eq. (2) in the space domain as a sum of the signals emitted from all the array elements as follows

$$S_{i,j} = \sum_p A_p \cos(kx_p^{i,j} - \varphi_p) \quad (3)$$

where  $k$  denotes the wavenumber, and  $x_p^{i,j}$  is the distance between pixel  $S_{i,j}$  and the element  $p$ . The value of the wavenumber  $k$  has to be calculated from the dispersion curves for the frequency at which the  $K^*K$  matrix was diagonalized.

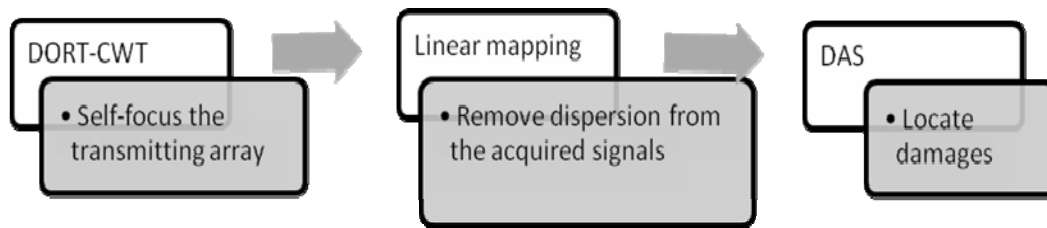


Figure 1. Flowchart of the damage detection algorithm.

### Concept of the damage detection method

Since it is possible to self focus an array on a target, a damage detection system based on a 2D array was developed. The array used in the system consists of two sub-arrays. The first of them operates in the transmission mode whereas the other operates in the reception mode. The DORT-CWT method used for self-focusing the transmitting array operates according to the flowchart presented in Figure 1, which illustrates the proposed damage detection algorithm. Due to focusing the target illuminated by the beam produces greater reflection than that would be obtained for a single transducer or unfocused array. The echoes are captured by the sub-array operating in reception mode and a linear mapping technique [12] is applied to the signals to remove the dispersion. In the final step damage imaging is performed using standard delay and sum (DAS) beamforming.

### EXPERIMENTAL VALIDATION

Two series of experiments will be presented in this section. The aim of the first experiment was to examine the performance of the DORT-CWT method applied to a linear ultrasonic array. In the first step, which was the same for all conducted experiments, the interelement responses for the array elements were recorded. Next, the backpropagated field was generated numerically. Finally, the array operating in transmission mode and generating signals defined by the DORT-CWT algorithm was used. The wave propagation in the plate was investigated with a use of laser scanning vibrometer.

The second series of experiments was conducted to investigate the operation of the damage detection system based on the self-focusing 2D array. Two array topologies were examined; the first was a cross-shaped array consisting of a linear transmitting sub-array and a linear receiving sub-array intersecting at an angle of  $90^\circ$ , and the second was a star-shaped array with four arms.

#### Experimental setup

The experiments were conducted using an aluminum plate with dimensions  $1000 \times 1000 \times 2$  mm shown in Figure 2. The plate was provided with three targets simulating damages: a  $10 \times 1$  mm notch, denoted as N, and two small masses (M1, M2) that were acoustically coupled to the investigated structure using wax. In the first series of experiments a linear array consisting of 8 CMAP12 PZT square multilayered transducers from Noliac, Denmark, was attached with wax in the middle of the plate. The transducers' size was  $2 \times 2 \times 2$  mm and their spacing was 5 mm.

The number of elements in the array was limited to 8 by the hardware used. A basic version of PAQ 16000 with PAS 8000 from EC Electronics, Poland was used for signal generation and data acquisition. Bursts of 2 cycles of 100 kHz sinusoid modulated with Hanning window were used to obtain inter-element responses. Data acquisition was performed with sampling frequency 2.5 MHz. The Noliac actuators used for the examined plate excited an enhanced asymmetric mode  $A_0$ , and a reduced, almost negligible symmetric mode  $S_0$ . Wavelength of the dominant mode at the excitation frequency 100 kHz was 12,9 mm.

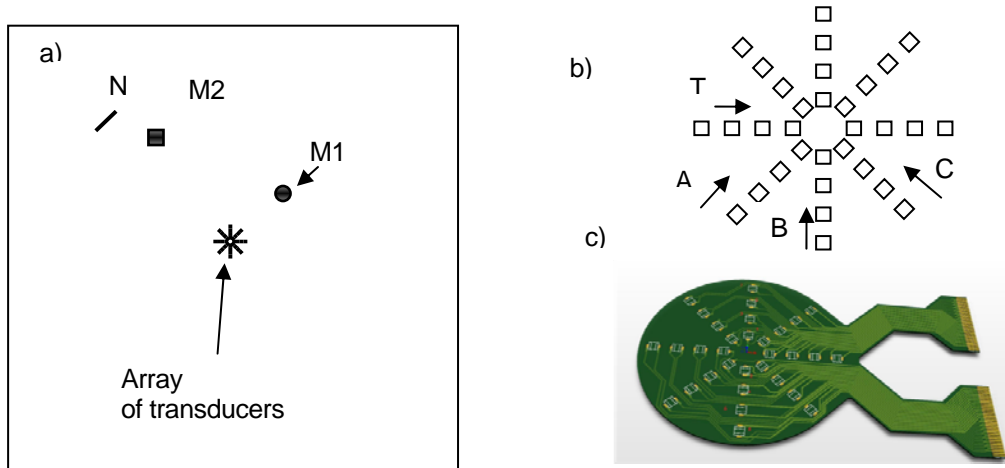


Figure 2. a) Scatterers distribution on the investigated plate. b) Topology of the star-shaped array, c) Laminate board used to build the array.

In the second series of experiments the linear array was replaced with the 2D array which topology shown in Figure 2b. The array, consisted of 4 linear sub-arrays intersecting at an angle of  $45^\circ$ ; each of them consisted of 8 CMAP12 PZTs spaced at a distance of 5 mm. The central element of each sub-array was left empty. The array, shown in Figure 2c, was made of a thin elastic printed circuit copper laminate which minimally influenced stiffness of the investigated plate.

The sub-array, denoted in Figure 2b as T, was used to collect the interelement impulse responses. Parameters of the excitation signal and data acquisition were kept constant in all experiments. The acquired signals were used to perform damage detection according to the flowchart presented in Figure 1.

### Self-focusing using linear array

In the first experiment the interelement responses were captured by the array and the acquired signals were processed using the DORT-CWT algorithm. A time distribution of the eigenvalues can be seen in Figure 3a where the significant peaks are pointed by arrows. Note that the number of the peaks is equal to the number of the damages existing in the investigated structure.

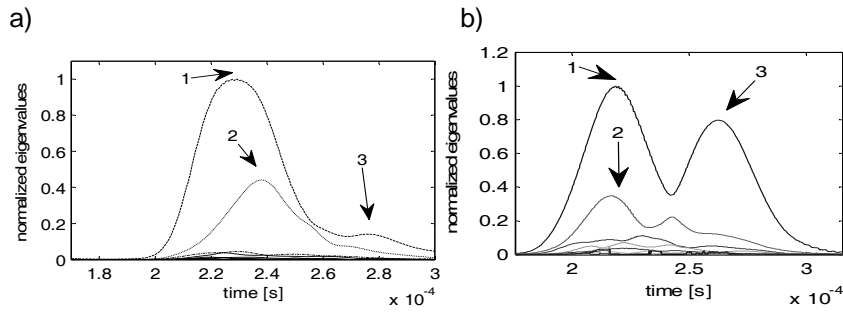


Figure 3. Normalized eigenvalues obtained with DORT-CWT for linear array (a) and for the sub-array of the star-shaped array (b).

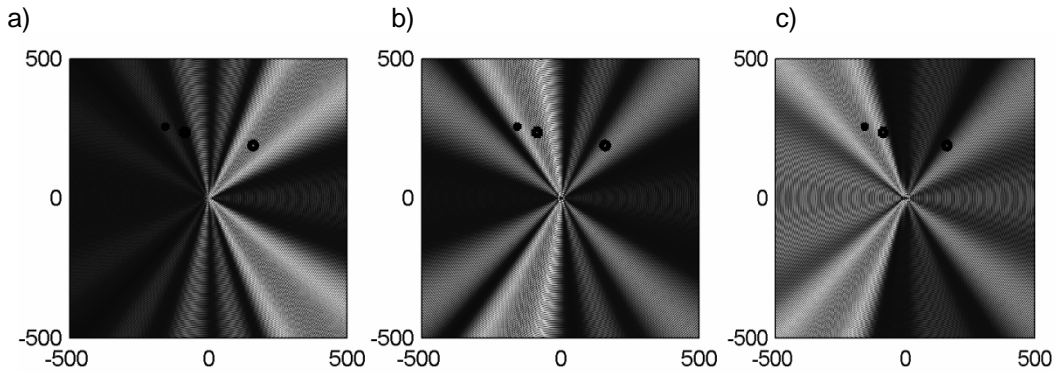


Figure 4. Numerical backpropagation. Wave focusing on the damages M1, M2 and N are shown in the panels a), b) and c), respectively. Markers indicate damages locations.

The eigenvectors corresponding to the significant peaks indicated in Figure 3a were numerically backpropagated and as it can be seen in Figure 4, all damages were correctly resolved, however, the backscattered fields are mirrored due to the linear topology of the array used in experiment.

Since the numerical backpropagation can give only a coarse information on the waves propagation the second experiment was conducted, which consisted in monitoring of the physical backpropagation in the plate using vibrometer.

In the experiment the array used previously to measure the interelement responses, was employed, in the transmission mode, to generate the phase-shifted signals. The phase shifts were computed for each array element according to eq. (2) using the distribution of eigenvalues presented in Figure 3a. Narrowband pulse signals were used to perform the backpropagation. In these experiments bursts of 4 cycles of a sine modulated with the Hanning window were generated. The center frequency, 100 kHz, was equal to that used in the  $K \times K$  matrix diagonalization. Scanning laser vibrometer Polytec PSV-400 was used for monitoring the field generated by the array. The vibrometer enabled contactless measurements of the out-of-plane surface velocity in the plate. Time signals were acquired in the points forming an arc-shaped grid. The maximum values of the first arriving wave packets captured by the scanning vibrometer were used to create the beampatterns shown in Figure 5. A high accuracy in the angle steering can be observed, which proves that the DORT-CWT algorithm correctly resolved the damages simulated in the plate.



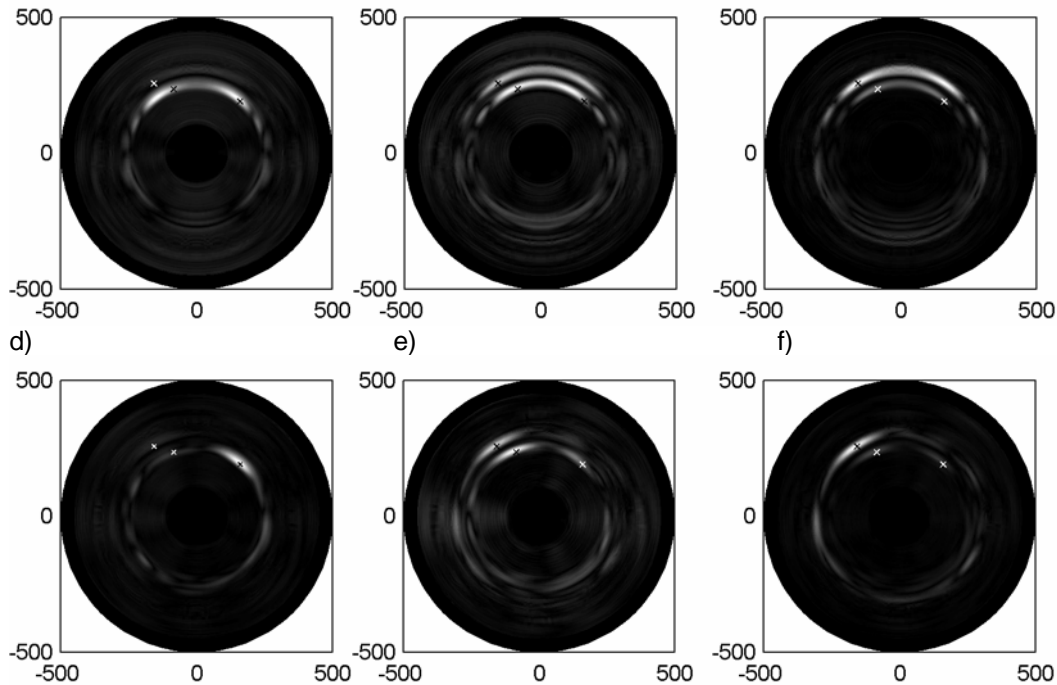


Figure 6. Damage imaging results obtained with the cross (a, b, c) and star-shaped array (d, e, f) for the transmitting sub-array steered in the direction of target denoted by M1, M2, and N.

In the next step the calculation was repeated using the data acquired by all three arms of the receiving sub-array. Subsequently, the dispersion removal was applied to the snapshots and damage imaging was performed according to the flowchart presented in Figure 1. From the results presented in Figure 6d, e and f can be seen that the damages were localized correctly. Moreover, in all images it can be observed that the illuminated targets are the strongest ones, which proves that the beams were steered correctly.

## CONCLUSIONS

An extension of the DORT method with application for self-focusing arrays for Lamb waves was presented in the paper. The performance of the modified method was confirmed experimentally showing a high accuracy in beam-steering.

The presented method was applied to a SHM system employing a 2D array operating in the pulse-echo mode. The developed system enabled damage detection and localization in an aluminum plate with superior reflection of damage illuminated by the self-focused beam. The shape of the designed array enabled avoiding the effect of ambiguous mirrored image.

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