Reduced-rank Channel Estimation and Tracking in Time-slotted CDMA Systems

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Abstract— **This paper investigates the estimation and tracking of time varying propagation channels in the uplink of a time slotted CDMA system. An antenna array is adopted at the base station. Both the estimation and tracking are performed by exploiting the low-rank nature of the channel matrix. The accuracy of the estimate is improved by using a multi-slot approach: the slowly varying component of the low-rank channel is estimated from the observation of successive midambles (inter-slot tracking), while the fast varying component is updated over the burst interval in decision-directed mode (intra-slot tracking).**

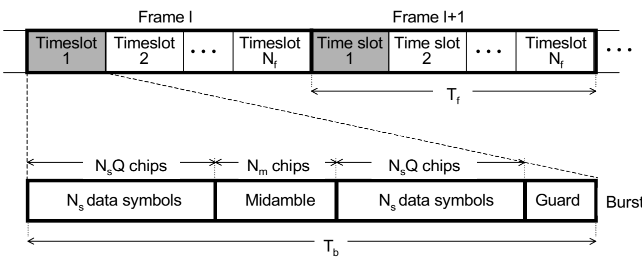

Fig. 1. Frame and burst structure for time-slotted CDMA systems.

I. Introduction

Multiuser detectors that combine signals from multiple antennas are powerful tools in CDMA systems. Since the number of parameters to be estimated grows with the number of antennas, an appropriate strategy to cope with a large number of channel unknowns and limited-length training data becomes mandatory. For a time-slotted CDMA system, we propose to improve the estimation accuracy by reducing the number of parameters that describe the channel matrix and by extending the training set with detected symbols. These two tools are combined in a method of reasonable computational complexity that also allows the tracking of the temporal variability within time slots caused by rapidly moving mobiles.

The reduced rank (RR) channel estimation methods provide a parsimonious parametrization of the channel matrix. The maximum likelihood solution proposed in [1] for the estimation of a RR linear regression has been applied to single-user [2]-[3] and multi-user [4] channel estimation in mobile communication systems. Since in time-slotted systems the accuracy of the channel estimate can be improved by exploiting training data from multiple slots [5], here we propose to extend the multi-user RR estimate [4] to multi-slot observations. In multipath propagation the amplitudes of the paths change rapidly due to fast fading, while their angles and delays will be fairly constant over several time slots for reasonable geometries and speeds. The low-rank signal subspace for each user will therefore be slowly varying and it can be estimated from the training sequences of successive slots (inter-slot estimation).

If the velocity of the mobile is large, then the channel fading within the burst cannot be neglected and an adaptive estimator is needed (intra-slot tracking) [6]. Starting from the estimate obtained from training data, the fast varying components of the low-rank channel are tracked over the burst interval in decision directed mode, using a sliding window multiuser detector (SWD) [7]. We propose a way of tracking only the fast varying component of the reduced-rank channel, which reduces the parameter space and improves the accuracy.

Notation: Lowercase (uppercase) bold symbols denote vectors (matrices), $(.)^*$ is the complex conjugate. For matrices, $(.)^T$ is matrix transpose, $(.)^H$ is the Hermitian transpose, $||.||^2$ is the Frobenius norm, $\mathbf{A} = \mathbf{A}^{H/2}\mathbf{A}^{1/2}$ is the Cholesky decomposition of the $M \times M$ matrix $\mathbf{A}$, $\mathrm{eig}_r\{\mathbf{A}\}$ denotes the $M \times r$ matrix of the r leading eigenvectors of $\mathbf{A}$, $\mathbf{a} = \mathrm{vec}\{\mathbf{A}\}$ is the stacking operator, and $\otimes$ is the Kronecker product. We will use three different discrete time indexes: the chip time t, the symbol time i and the frame index ℓ.

II. Reduced-rank Channel Models

A time slotted CDMA mobile radio system uses a combination of TDMA (time division multiple access) and CDMA (code division multiple access) to separate the users in the time and/or in the code domain. The frame structure of the TDMA component is illustrated in Fig. 1. Each frame has duration T_f and is subdivided into N_f slots. An uplink is considered where K mobile users are active in each time slot. They use the same frequency band but different spreading codes, $\mathbf{c}_k = [c_k(1) \cdots c_k(Q)]^T$ for the kth user (Q is the spreading factor). During the time slot each user transmits a burst of duration $T_b = T_f/N_f$ consisting of two data blocks, a user specific training sequence (midamble) and a guard period. The known training sequence consists of $N_m = N + W - 1$ chips, where W is the channel length in chip intervals T_c and N is the number of chips used for channel estimation. Each data block contains N_s QPSK symbols of duration $T_s = QT_c$. The time interval between two successive slots of the same user is T_f.

The discrete-time model of the uplink should describe the signals received by the M antennas/radio access points, sampled at the chip rate $1/T_c$ after chip matched filtering. The time-varying baseband channel between the kth mobile transmitter and the receivers can be described by the $M \times W$ space-time matrix $\mathbf{H}_k(t) = [\mathbf{h}_{k,1}(t) \cdots \mathbf{h}_{k,M}(t)]^T$. Here the vector $\mathbf{h}_{k,m}(t)$ represents the discrete-time channel impulse response for the link between the kth user and the mth antenna. The propagation channel is modelled by the superposition of P_k paths, each characterized by the direction of arrival ($\vartheta_{k,p}$), the delay ($\tau_{k,p}$) and the complex valued amplitude ($\alpha_{k,p}(t)$):

$$\mathbf{H}_k(t) = \sum_{p=1}^{P_k} \alpha_{k,p}(t)\mathbf{a}(\vartheta_{k,p})\mathbf{g}(\tau_{k,p})^T. \qquad (1)$$

The $W \times 1$ vector $\mathbf{g}(\tau_{k,p})$ represents the chip pulse shape delayed by $\tau_{k,p}$ and $\mathbf{a}(\vartheta_{k,p})$ is the $M \times 1$ array gain for $\vartheta_{k,p}$. The angles and the delays vary slowly and can therefore be assumed to be invariant over L time slots, if L is not too large. The fading amplitudes $\alpha_{k,p}(t)$ change much more rapidly due to the movement of the mobile and are assumed to be uncorrelated from slot to slot.

Let $\mathbf{R}_{S,k} = \mathrm{E}_\alpha[\mathbf{H}_k(t)\mathbf{H}_k(t)^H]$ denote the spatial correlation matrix of the channel. Its rank-order $r_{S,k}$ corresponds to the number of the discernible angles. The number of the resolvable delays is equal to the rank-order $r_{T,k}$ of the temporal correlation matrix $\mathbf{R}_{T,k} = \mathrm{E}_\alpha[\mathbf{H}_k(t)^H\mathbf{H}_k(t)]$. The channel $\mathbf{H}_k(t)$ lies in the subspace spanned by either the $r_{S,k}$ leading eigenvectors of $\mathbf{R}_{S,k}$ or the $r_{T,k}$ leading eigenvectors of $\mathbf{R}_{T,k}$. Therefore we propose to calculate the signal subspace by estimating the correlation matrices from the observation of L successive slots and then project the least squares estimate of $\mathbf{H}_k(t)$ onto the corresponding subspaces.

This is done by extending the reduced-rank model [1] to multiple slots. The model structure for $\mathbf{H}_k(t)$ will be constrained to have low rank $r_k = \min(r_{S,k}, r_{T,k}) < \min(M, W)$. It is represented as the product of a $M \times r_k$ space (S) component $\mathbf{A}_k(t)$ and a $W \times r_k$ time (T) component $\mathbf{B}_k(t)$. Two alternative models will be used to describe the temporal variability of the low-rank channel matrix:

$$\text{Model S-RR} : \quad \mathbf{H}_k(t) = \mathbf{A}_k\mathbf{B}_k^H(t), \tag{2}$$

$$\text{Model T-RR} : \quad \mathbf{H}_k(t) = \mathbf{A}_k(t)\mathbf{B}_k^H. \tag{3}$$

In the S-RR model the S-component remains constant, whereas the T-component varies over the bursts. The opposite holds in the T-RR case.

The model S-RR is appropriate when $r_{S,k} < r_{T,k}$. For instance, if there is no angular spread ($\vartheta_{k,p} = \vartheta_k \ \forall p$) the rank order is $r_k = 1$ and the channel can be expressed as $\mathbf{H}_k(t) = \mathbf{a}_k\mathbf{b}_k^H(t)$, where $\mathbf{b}_k(t) = \sum_{p=1}^{P_k}\alpha_{k,p}(t)\mathbf{g}(\tau_{k,p})$ and $\mathbf{a}_k = \mathbf{a}(\vartheta_k)$. More generally, the channel may be composed of $r_k > 1$ clusters, each comprising paths having similar angles but different delays (the attributes "different" and "similar" are to be understood with respect to the resolution of the antenna array and to the system bandwidth). The rank-order of the channel matrix can then be approximated by the number of the clusters, $r_k = r_{S,k}$. A similar reasoning applies to the T-RR model, where the rank is determined by the temporal pattern (the number of the resolvable delays), $r_k = r_{T,k}$.

III. Inter-slot Channel Estimation

Consider the model for the signals received by the antenna array during the ℓth midamble, $\ell = 1, ..., L$. The channel matrix of the kth user is assumed to be constant during the training period and is denoted by $\mathbf{H}_k(\ell)$. The estimate of the K channels is based on the transmission of K different training sequences. Each training sequence is arranged in a $W \times N$ Toeplitz matrix $\mathbf{X}_k$ to represent the operation of convolution with the channel. Let $\mathbf{Y}(\ell)$ be the $M \times N$ matrix containing N samples of the signals received by the M antennas. The multiuser matrices $\mathbf{H}(\ell) = [\mathbf{H}_1(\ell)\cdots\mathbf{H}_K(\ell)]$ and $\mathbf{X} = [\mathbf{X}_1^T\cdots\mathbf{X}_K^T]^T$ include, respectively, the K channels and the K training sequence matrices. The signals received during the L midambles can now be represented by

$$\mathbf{Y}(\ell) = \sum_{k=1}^{K}\mathbf{H}_k(\ell)\mathbf{X}_k + \mathbf{N}(\ell) = \mathbf{H}(\ell)\mathbf{X} + \mathbf{N}(\ell),\ \ell = 1, \ldots, L, \tag{4}$$

where $\mathbf{N}(\ell) = [\mathbf{n}_\ell(1)\cdots\mathbf{n}_\ell(N)]$ represents both the additive ambient noise and the inter-cell interference. The noise is assumed temporally uncorrelated and spatially correlated, $\mathbf{n}_\ell(t) \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_n)$, with $[\mathbf{R}_n]_{m,m} = \sigma^2$ being the variance at each antenna element.

The unconstrained (or full rank, FR) maximum likelihood (ML) estimate of the channels $\forall\ell$ and the covariance matrix of noise are given by [4]

$$\hat{\mathbf{H}}(\ell) = [\hat{\mathbf{H}}_1(\ell)\cdots\hat{\mathbf{H}}_K(\ell)] = \mathbf{Y}(\ell)\mathbf{X}^H(\mathbf{X}\mathbf{X}^H)^{-1}, \tag{5}$$

$$\hat{\mathbf{R}}_n = \frac{1}{LN}\sum_{\ell=1}^{L}[\mathbf{Y}(\ell) - \hat{\mathbf{H}}(\ell)\mathbf{X}][\mathbf{Y}(\ell) - \hat{\mathbf{H}}(\ell)\mathbf{X}]^H. \tag{6}$$

If the rank of the channel matrix is smaller than $\min(M, W)$, the use of a reduced-rank model (2)-(3) is advantageous for the reduction of the unknown parameters. Here the RR estimation is extended to multi-slot observations in order to further increase the accuracy of the channel estimate. The models (2)-(3) lead to different RR algorithms: multi slot RR estimation in the space domain (S-MS-RR) and in the time domain (T-MS-RR) respectively. A third estimation method is obtained as a combination of the two algorithms (ST-MS-RR).

In the following sections the RR multi-slot estimates are obtained from the ensemble of the L whitened FR estimates denoted by $\tilde{\mathbf{H}}(\ell) = [\tilde{\mathbf{H}}_1(\ell)\cdots\tilde{\mathbf{H}}_K(\ell)]$, where $\tilde{\mathbf{H}}_k(\ell)$ is

$$\tilde{\mathbf{H}}_k(\ell) = \hat{\mathbf{R}}_n^{-H/2}\hat{\mathbf{H}}_k(\ell)\mathbf{R}_{xx,k}^{H/2}, \tag{7}$$

and $\mathbf{R}_{xx,k} = \mathbf{X}_k\mathbf{X}_k^H/N$. The whitening is carried out by assuming that the correlation between the training sequences of different users is negligible.

A. RR Estimate in the Space Domain (S-MS-RR)

The channel model (2) assumes an invariant S-component for the channel, while the T-component changes from frame to frame. Let us define for the kth user the estimate of the spatial correlation matrix of the whitened channel estimate:

$$\hat{\tilde{\mathbf{R}}}_{S,k} = \frac{1}{L}\sum_{\ell=1}^{L}\tilde{\mathbf{H}}_k(\ell)\tilde{\mathbf{H}}_k^H(\ell). \tag{8}$$

According to the system model (4), it can be shown that the multi-slot ML estimation of the channels and the covariance of the noise under the constraint $\mathbf{H}_k(\ell) = \mathbf{A}_k\mathbf{B}_k^H(\ell), \forall k, \ell$, yields the solution (see [8] for single-user estimate):

$$\hat{\mathbf{H}}_{\mathrm{RR},k}(\ell) = \hat{\mathbf{A}}_k\hat{\mathbf{B}}_k^H(\ell) = \hat{\mathbf{R}}_n^{H/2}\mathbf{\Pi}_{S,k}\tilde{\mathbf{H}}_k(\ell)\mathbf{R}_{xx,k}^{-H/2}, \tag{9}$$

where $\hat{\mathbf{R}}_n$ is defined in (6) and $\mathbf{\Pi}_{S,k} = \mathbf{U}_S\mathbf{U}_S^H$ is the projector onto the kth signal subspace. The latter is spanned by the r_k leading eigenvectors of $\hat{\hat{\mathbf{R}}}_{S,k}$: $\mathbf{U}_S = \mathrm{eig}_{r_k}\{\hat{\hat{\mathbf{R}}}_{S,k}\}$. The noise covariance matrix is estimated from the residuals of the channel estimation $\hat{\mathbf{N}}(\ell) = \mathbf{Y}(\ell) - \sum_{k=1}^{K} \hat{\mathbf{H}}_{\mathrm{RR},k}(\ell)\mathbf{X}_k$, as: $\hat{\mathbf{R}}_{n,\mathrm{RR}} = \sum_{\ell=1}^{L} \hat{\mathbf{N}}(\ell)\hat{\mathbf{N}}^H(\ell)/LN$.

B. RR Estimate in the Time Domain (T-MS-RR)

This variant assumes the model (3). The temporal correlation matrix of the whitened channel estimate for the user k is:

$$\hat{\hat{\mathbf{R}}}_{T,k} = \frac{1}{L}\sum_{\ell=1}^{L} \tilde{\mathbf{H}}_k^H(\ell)\tilde{\mathbf{H}}_k(\ell). \tag{10}$$

Similarly to (9) the multi-slot ML estimate under the constraint $\mathbf{H}_k(\ell) = \mathbf{A}_k(\ell)\mathbf{B}_k^H$, $\forall k, \ell$, is given by [4]

$$\hat{\mathbf{H}}_{\mathrm{RR},k}(\ell) = \hat{\mathbf{A}}_k(\ell)\hat{\mathbf{B}}_k^H = \hat{\mathbf{R}}_n^{H/2}\,\tilde{\mathbf{H}}_k(\ell)\,\mathbf{\Pi}_{T,k}\,\mathbf{R}_{xx,k}^{-H/2}. \tag{11}$$

Here $\mathbf{\Pi}_{T,k} = \mathbf{U}_T\mathbf{U}_T^H$ is the projector onto the signal subspace estimated for the kth user. The projector is formed by the eigenvectors $\mathbf{U}_T = \mathrm{eig}_{r_k}\{\hat{\hat{\mathbf{R}}}_{T,k}\}$ associated to the r_k largest eigenvalues. The noise covariance matrix is estimated as in Section III-B.

C. RR Estimate in Space and Time Domain (ST-MS-RR)

The channel estimate can be obtained by combining the methods S-MS-RR and T-MS-RR and estimating the signal subspace in both the temporal and the spatial domain [4], [5]:

$$\hat{\mathbf{H}}_{\mathrm{RR},k}(\ell) = \hat{\mathbf{R}}_n^{H/2}\mathbf{\Pi}_{S,k}\tilde{\mathbf{H}}_k(\ell)\,\mathbf{\Pi}_{T,k}\,\mathbf{R}_{xx,k}^{-H/2}. \tag{12}$$

Here $\mathbf{\Pi}_{S,k}$ ($\mathbf{\Pi}_{T,k}$) is the projector onto the space spanned by the $r_{S,k}$ ($r_{T,k}$) leading eigenvectors of the spatial correlation matrix (8) (temporal correlation matrix (10)). In general $r_{S,k} \neq r_{T,k}$.

IV. Intra-slot Channel Tracking

For mobile users moving at high speed, the coefficients of the matrices $\mathbf{H}_k(t)$ vary significantly and an adaptive algorithm must be adopted to track the channel variations. The RR estimate can then be obtained as follows. First the training sequences are used as described in the Section III. The estimate so obtained is then adapted recursively within each slot using the decided symbols as regressors.

We will investigate two different tracking approaches. The first method performs an update of the unconstrained estimate of $\mathbf{H}_k(t)$ and then reduces the rank by projecting the updated channel estimate onto the invariant signal subspace. The second method exploits the low-rank property while tracking by adjusting only the fast varying component of the channel.

A. System Model

Consider the received signal $\{\mathbf{y}(t)\}$ generated by the transmission of one data block of N_s symbols $\{d_k(i)\}$ (the slot index is dropped in this section as it is not significant)

$$\mathbf{y}(t) = \sum_{k=1}^{K} \mathbf{H}_k(t)\mathbf{x}_k(t) + \mathbf{n}(t), \tag{13}$$

where t is the discrete chip time and $\mathbf{y}(t)$ is the $M \times 1$ vector containing the tth sample of the received signals. The elements $\{x_k(t)\}$ of the vector $\mathbf{x}_k(t) = [x_k(t)\cdots x_k(t-W+1)]^T$ are obtained in the decision-directed mode by spreading the estimated symbols $\{d_k(i)\}$ with the code $\mathbf{c}_k$. If the channel does not vary significantly during the symbol interval, a discrete time model at the symbol rate can represent Q samples of the received signals (13):

$$\mathbf{Y}(i) = \sum_{k=1}^{K} \mathbf{H}_k(i)\mathbf{X}_k(i) + \mathbf{N}(i). \tag{14}$$

Here i denotes the discrete symbol time, while $\mathbf{Y}(i)$ and $\mathbf{X}_k(i)$ are matrices containing the Q snapshots of the received and transmitted signals.

The tracking algorithms discussed in this Section are confined to the channel model (2) and are only applied forward in time from the midamble based estimate calculated as in (9).

B. Unconstrained Tracking

A simple adaptive method is obtained by considering the model (14) without any constraint on the channel matrix:

$$\mathbf{Y}(i) = \mathbf{H}(i)\mathbf{X}(i) + \mathbf{N}(i), \tag{15}$$

here $\mathbf{H}(i) = [\mathbf{H}_1(i)\cdots\mathbf{H}_K(i)]$, $\mathbf{X}(i) = [\mathbf{X}_1^T(i)\cdots\mathbf{X}_K^T(i)]^T$. The unconstrained channel estimate $\hat{\mathbf{H}}(i)$ can be adapted from (15) by a recursive algorithm such as recursive least squares (RLS) or least mean squares (LMS) [9]. The RR estimate for the kth user is then obtained from (9) by projecting the updated estimate $\hat{\mathbf{H}}_k(i)$ onto the signal subspace:

$$\hat{\mathbf{H}}_{\mathrm{RR},k}(i) = \hat{\mathbf{R}}_n^{H/2}\mathbf{\Pi}_{S,k}\hat{\mathbf{R}}_n^{-H/2}\hat{\mathbf{H}}_k(i). \tag{16}$$

The projector $\mathbf{\Pi}_{S,k}$ and the covariance matrix $\hat{\mathbf{R}}_n$ are obtained by the multi-slot estimation described in Section III and remain unchanged over the burst interval. As an example we consider a block RLS algorithm with exponential forgetting factor λ [9]. If the velocity for all the mobiles is very low, then the forgetting factor could be set very close to $\lambda = 1$ to virtually extend the training sequence and thus improve the performance of the channel estimate (bootstrap estimation method). Indeed, the accuracy depends on the length of the training data and will be improved by a factor N_sQ/N for large signal to noise ratios.

C. Reduced Rank Tracking

By assuming the S-RR channel model (2), the data model (14) for the adaptive channel estimator reduces to

$$\mathbf{Y}(i) = \sum_{k=1}^{K} \mathbf{A}_k\mathbf{B}_k^H(i)\mathbf{X}_k(i) + \mathbf{N}(i). \tag{17}$$

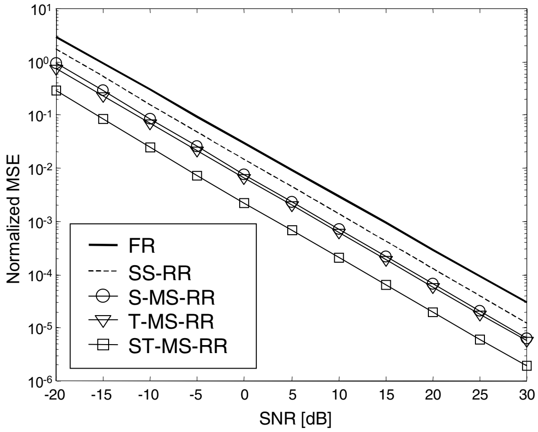

Fig. 2. Comparison of the normalized MSE for single-slot (SS) and multi-slot (MS) reduced rank channel estimation (L=20).

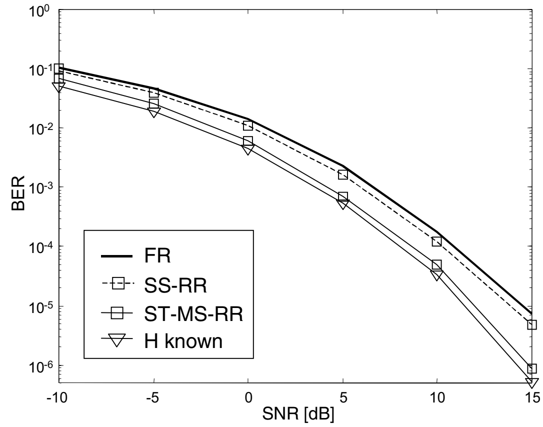

Fig. 3. Comparison of the performance of single-slot (SS) and space-time multi-slot (ST-MS) channel estimation (L=10). The channels are time invariant over the burst interval.

In order to convert (17) into a conventional linear regression model, the data collected at the ith instant is stacked into the $MQ \times 1$ vector $\mathbf{y}(i) = \mathrm{vec}\{\mathbf{Y}(i)\}$. Using the property $\mathrm{vec}\{\mathbf{ABC}\} = (\mathbf{C}^T \otimes \mathbf{A})\,\mathrm{vec}\{\mathbf{B}\}$, we obtain:

$$\mathbf{y}(i) = \sum_{k=1}^{K} \mathbf{\Psi}_k^H(i)\mathbf{b}_k(i) + \mathbf{n}(i) = \mathbf{\Psi}^H(i)\mathbf{b}(i) + \mathbf{n}(i), \quad (18)$$

where $\mathbf{n}(i)$= $\mathrm{vec}\{\mathbf{N}(i)\}$ is the noise vector, the $Wr_k \times QM$ regressor matrix $\mathbf{\Psi}_k(i) = \mathbf{X}_k^*(i) \otimes \mathbf{A}_k^H$ is known (as $\mathbf{A}_k$ is estimated from multi-slot) and $\mathbf{b}_k(i) = \mathrm{vec}\{\mathbf{B}_k^H(i)\}$ is the parameter vector of length Wr_k for the kth user. The model (18) is a linear regression, where $\mathbf{\Psi}(i)$= $[\mathbf{\Psi}_1^H(i) \cdots \mathbf{\Psi}_K^H(i)]^H$ denotes the regression matrix of dimension $Wr \times QM$, with $r = \sum_{k=1}^{K} r_k$, and $\mathbf{b}(i)$= $[\mathbf{b}_1(i) \cdots \mathbf{b}_K(i)]$ is the overall parameter vector of length Wr.

The time varying parameter vector $\mathbf{b}(i)$ can be estimated by standard recursive algorithms having the structure

$$\varepsilon(i) = \mathbf{y}(i) - \mathbf{\Psi}^H(i)\hat{\mathbf{b}}(i), \quad (19)$$
$$\hat{\mathbf{b}}(i+1) = \hat{\mathbf{b}}(i) + \mathbf{\Gamma}(i)\mathbf{\Psi}(i)\varepsilon(i). \quad (20)$$

Above $\varepsilon(i)$ is the prediction error and $\mathbf{\Gamma}(i)$ is a positive definite matrix. The channel estimate is obtained from the updated parameter vector $\hat{\mathbf{b}}_k(i+1) = \mathrm{vec}\{\hat{\mathbf{B}}_k(i+1)\}$ as

$$\hat{\mathbf{H}}_{\mathrm{RR},k}(i+1) = \hat{\mathbf{A}}_k \hat{\mathbf{B}}_k^H(i+1). \quad (21)$$

The LMS algorithm corresponds to (19)-(20) with the gain matrix $\mathbf{\Gamma}(i) = \mu(i)\mathbf{I}_{rW}$, where $\mu(i)$ is the time-varying gain factor. An approach to improve the adaptation performance is to use a time varying step length $\mu(i)$ that decreases with time to a stationary value μ_0. Furthermore, μ_0 could be optimized with respect to the mobile velocity and the signal to noise ratio (SNR). A large SNR allows a larger step length and faster tracking [6]. Similarly to the RLS algorithm described in the previous section, if the mobile is moving very slowly, μ_0 should be chosen close to zero so that the method reduces to a bootstrap channel estimation technique. As users can have different velocities and different SNRs, a user-specific step length can also be introduced by defining the gain matrix as $\mathbf{\Gamma}(i) = \mathrm{diag}\{\mu_1(i)\mathbf{I}_{r_1 W}, \ldots, \mu_K(i)\mathbf{I}_{r_K W}\}$.

V. Simulation Results

In the examples below the performance of the adaptive RR channel estimation is evaluated by simulating the uplink of the UTRA-TDD standard. The numerical results refer to a linear antenna array of $M = 8$ omnidirectional antennas half-wavelength spaced apart, with $K = 8$ users being active in all time slots of duration $T_s = 666\ \mu$s. Each data block contains $N_s = 61$ QPSK symbols, spread by Hadamard codes having spreading factor $Q = 16$. The training sequences are chosen according to the standard specifications with length $N_m = 512$ and their correlation properties are such that $\left\|\mathbf{X}_k\mathbf{X}_h^H\right\| << \left\|\mathbf{X}_k\mathbf{X}_k^H\right\|$, for $k \neq h$. The TDD-UTRA system operates at approximately 2 GHz with a frame duration of $T_f = 10$ ms. If the velocity of the mobile user is $v \geq$50 km/h, the fading will be almost uncorrelated between slots in different frames. Moreover, the assumption of stationary angles and delays seems reasonable for $L \leq 20$ slots, as a MS with velocity $v \leq 200$ km/h moves less than 11m. This motivates the use of a multi-slot approach for channel estimation.

A three-path propagation channel is simulated for each user. The path angles are random variables with distribution $\vartheta_{k,p} \sim \mathcal{N}(\vartheta_k, \sigma_\vartheta^2)$ with $\vartheta_k \sim \mathcal{U}[-\pi/3, \pi/3]$ and standard deviation $\sigma_\vartheta = \pi/36$. The delays are fixed to $\tau_{k,1} = 0$, $\tau_{k,2} = 0.73T_c$, $\tau_{k,3} = 5.82T_c$ and the channel-length is $W = 13$. The complex amplitudes are Rayleigh distributed, $\alpha_{k,p}(t) \sim \mathcal{CN}(0, \alpha_p^2)$, with α_1^2 =0.28, α_2^2 =0.58, α_3^2 =0.14. Each time varying amplitude is simulated as a random process with Jakes power spectral density function depending on the Doppler frequency f_D. The rank of the channel is $r_k \leq 3$ for each user. The same holds for the rank of both the spatial and the temporal component, i.e. $r_S \leq 3$ and $r_T \leq 3$. In all the simulations the Gaussian noise is spatially correlated due to an interferer with direction of arrival $\pi/6$: $[\mathbf{R}_n]_{m,l} = \sigma^2\{0.9\exp(-i\pi\sin(\pi/6))\}^{l-m}$.

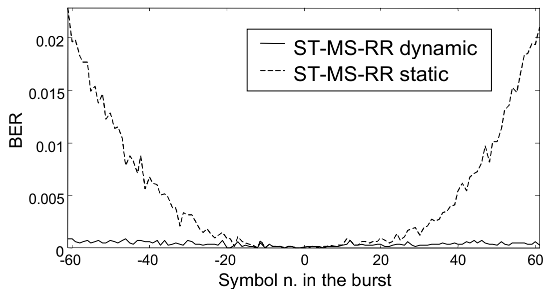

Fig. 4. BER for varying position of the symbol in the burst for SNR=15dB and v=200 km/h.

Figure 2 compares the performance of the FR estimate with the single-slot (SS) and multi-slot (MS) RR estimates with $L = 20$. The normalized MSE $||\Delta\mathbf{H}||^2/\,||\mathbf{H}||^2$ is evaluated for varying signal to noise ratio defined as $\mathrm{SNR} = \mathrm{E}[(||\mathbf{h}_{k,m}||^2)]/\sigma^2$. The RR estimates that assume rank 3 outperform the full-rank estimate for all the SNR values. With respect to the SS-RR, the gain of S-MS-RR and T-MS-RR is approximately 3 dB in SNR and the accuracy is further improved when both the spatial and temporal projections are used in ST-MS-RR, leading to an overall gain of 8 dB.

In Fig. 3 the SS-RR and the ST-MS-RR with $L = 20$ are evaluated in terms of BER for uncoded bits vs. $\mathrm{SNR} = \mathrm{E_{bit}}\,/N_0$. The fading is assumed uncorrelated from frame to frame and the channel is static over the whole burst interval. The data symbols are estimated by a sliding window MMSE multiuser detector [7]. The multi-slot RR estimate outperforms both the FR and the single–slot RR estimates and approaches the performance obtained with known channels.

For moderate fading, such as $f_D T_s N_s = 0.1$ considered in Fig. 4 ($v = 200$ km/h), the estimate obtained from the training sequences can not be used within the whole slot as the error probability increases at the ends of the burst. The adaptation of the channel estimator is considered in Fig. 5. The performance of the static and dynamic MS-RR estimators are compared for $L = 10$. Two different algorithms based on the S-RR model are considered for the adaptation: a RLS unconstrained tracking and an LMS reduced-rank tracking. In both the methods the performance are improved by applying also the temporal projector $\mathbf{\Pi}_{T,k}$ to the updated channel matrix (21), as in (12). An MMSE-SWD is used to estimate the data symbols. The use of the constrained tracking guarantees lower BER at moderate and high SNR's compared to the unconstrained tracking.

The correlation matrix used in the multiuser detector changes over the burst interval as the correlation values depend on the convolution between the user signatures and the time varying channels. The update of the detector can be a computationally complex task due to the correlation matrix inversion, hence an efficient algorithm has to be used such as [7].

VI. Concluding remarks

An adaptive reduced-rank estimator for time varying propagation channels has been presented. The RR estimate is obtained from the training sequences of successive slots by constraining the spatial and/or temporal component of the low-rank channel matrix to remain unchanged. Then it is adapted recursively within each slot by using the estimated symbols. Simulations have indicated improvements in the system performance for both slow and fast time-varying channels.

In this preliminary investigation, standard recursive algorithms have been considered, in particular RLS and LMS. Further performance improvements can be obtained with suitably tuned Kalman trackers that use higher order models for the fading statistics, and models for parameter correlations. Since the number of parameters to be tracked will be rather high for large M, Kalman estimator will have a rather high computational complexity. However, adaptation laws can be constructed to provide close to Kalman performance at a computational complexity similar to that of LMS [6], [10]. The use of this method in our present problem will be further investigated in our continued work.

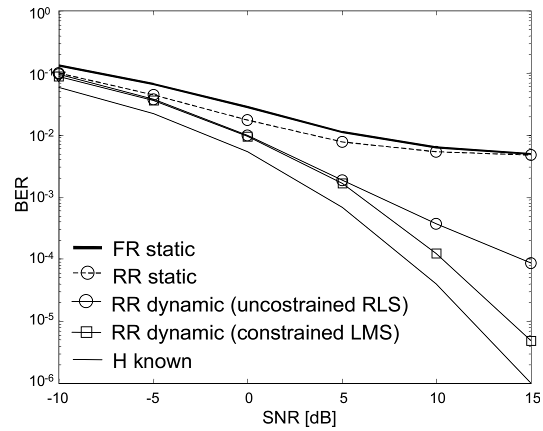

Fig. 5. Performance of static and dynamic multi-slot RR channel estimation for v=200km/h.

References

[1] P. Stoica and M. Viberg, "Maximum likelihood parameter and rank estimation in reduced-rank multivariate linear regression", *IEEE Trans. Signal Processing*, vol. 44, pp. 3069-3078, Dec. 1996.

[2] E. Lindskog and C. Tidestav, "Reduced rank channel estimation", *Proc. IEEE VTC Spring '99*, vol. 2, pp. 1126-1130, May 1999.

[3] D. Giancola, A. Sanguanini, and U. Spagnolini, "Variable rank receiver structures for low-rank space-time channels", *Proc. IEEE VTC Spring '99,* vol.1, pp. 65-69, May 1999.

[4] M. Nicoli, *Multi-user Reduced Rank Receivers for TD/CDMA Systems*, Ph.D. Thesis, Politecnico di Milano, Italy, Dec. 2001.

[5] O. Simeone and U. Spagnolini, "Multi-slot estimation of space-time channels", *Proc. IEEE ICC '02*, May 2002.

[6] L. Lindbom, M. Sternad, and A. Ahlén, "Tracking of time-varying mobile radio channels. Part I: the Wiener LMS algorithm", *IEEE Trans. Communications*, vol. 49, Dec. 2001.

[7] M. J. Juntti and B. Aazhang, "Iterative implementation of linear multiuser detection for dynamic asynchronous CDMA systems", *IEEE Trans. Communications,* vol. 46, no. 4, pp. 503-508, Apr. 1998.

[8] P. Forster and T. Asté, "Maximum likelihood multichannel estimation under reduced rank constraint", *Proc. IEEE ICASSP '98,* vol. 6, pp. 3317-3320, May 1998.

[9] S. Haykin, *Adaptive Filter Theory*, 3rd ed., Prentice Hall, Englewood Cliffs, NJ, 1996.

[10] A. Ahlén, M. Sternad, and L. Lindbom, "Iterative Wiener design of adaptation laws with constant gains", *Proc. IEEE ICASSP '01,* vol. 6, pp. 3861-3864, May 2001.