

Tracking of Time-varying Mobile Radio Channels with WLMS Algorithms: A Case Study on D-AMPS 1900 Channels

Mikael Sternad*, Lars Lindbom¹ and Anders Ahlén*

*Signals and Systems, Uppsala University, PO Box 528, SE-75120, Uppsala, Sweden.

¹Ericsson Infotech, PO Box 1038, SE-65115 Karlstad

ms@signal.uu.se, Lars.Lindbom@ein.ericsson.se, aa@signal.uu.se

Abstract - Low-complexity WLMS adaptation algorithms, of use for channel estimation, have been presented in a companion paper. Their use and design is here evaluated on the fast fading radio channels encountered in TDMA systems based on IS-136. An exact analytical expression for the tracking MSE on two-tap FIR channels is presented and utilized. With this tool, the MSE performance and robustness of WLMS algorithms based on different statistical models can be investigated. A simulation study compares the uncoded bit error rate of detectors, where channel trackers are used in decision directed mode in conjunction with Viterbi algorithms.

A Viterbi detector combined with WLMS, based on second order autoregressive fading models possibly combined with integration, provides good performance and robustness at a reasonable complexity.

I. Introduction and Outline

In D-AMPS 900 and 1900 (or IS-136) digital mobile TDMA systems, a relatively low symbol rate and long data slots (6.67 ms) cause severe fading. In 1900 MHz systems, one or two fading dips can be expected within each data slot. Furthermore, large variations in fading rates and frequency selectivity are encountered, so well designed channel estimators are crucial for obtaining acceptable performance in the presence of intersymbol interference. Channel estimates obtained from training sequences cannot be used over the whole frames and interpolation of channel estimates between training sequences provides inadequate performance. The same is true for the decision-directed LMS and RLS algorithms.

In a related contribution to VTC2000 [1], we have presented the WLMS algorithm, which can efficiently utilize the fading statistics. It enables the design of high-performance adaptation laws with LMS computational complexity for white regressors. An early design related to this class of algorithms [6] has been successfully applied to tracking problems in both the D-AMPS 900 and 1900MHz systems [2, 4, 11].

We will here investigate opportunities, design choices and possible problems when applying the WLMS algorithm in a realistic scenario.

With a two-tap fading channel and a symbol alphabet with constant modulus, an *exact* performance analysis can be performed. Analytical expressions for the mean square parameter tracking error are presented. The performance is investigated with this tool for fading rates for which the adaptation laws are tuned, as well as for other fading rates. The bit error rate performance is then evaluated by simulation, for adaptive Viterbi detectors in decision-directed mode.

II. The Channel Model

A symbol-spaced baseband mobile radio channel is assumed described by the time-varying linear regression

$$y_t = (u_t \dots u_{t-M+1}) \begin{pmatrix} h_{0,t} \\ \vdots \\ h_{M-1,t} \end{pmatrix} + v_t = \varphi_t^* h_t + v_t \quad (1)$$

where y_t is the received baseband signal, here assumed to be a scalar. The possibly multi-variable channel with M taps is represented by h_t and $\{u_t\}$ are transmitted symbols, with zero mean. The noise v_t has zero mean and a variance σ_v^2 . The autocorrelation matrix $E[\varphi_t \varphi_t^*] \triangleq \mathbf{R}$ of the regressor sequence $\{\varphi_t^*\}$ is known and nonsingular. When u_t is white, $\mathbf{R} = \sigma_u^2 \mathbf{I}$. In D-AMPS systems, $M = 1$ (flat fading) or $M = 2$.

The channel coefficients will be subject to fading characterized by the maximum Doppler frequency ω_D , which may not be perfectly known. For the purpose of our investigation, we shall use Jakes' fading model [3]. When the vehicle velocity is constant, the channel coefficients will then be stationary, circular Gaussian processes with zero means and covariance function

$$r_h(\ell) = E\{h_t h_{t-\ell}^*\} = \mathbf{R}_h J_0(\Omega_D \ell) \quad \ell = 0, \pm 1, \dots \quad (2)$$

which yields the classical fading spectrum

$$\phi_h(\Omega) = \begin{cases} \frac{2}{\sqrt{\Omega_D^2 - \Omega^2}} \mathbf{R}_h & |\Omega| < \Omega_D \\ 0 & |\Omega| > \Omega_D \end{cases} \quad (3)$$

Here, $\mathbf{R}_h = E\{h_t h_t^*\}$, $J_0(\cdot)$ denotes the Bessel function of the first kind and zero order and $\Omega = \omega T$, $\Omega_D = \omega_D T$. The symbol time T is 41.15 μ s in IS-136.

III. The Channel Estimator

A WLMS design begins with the selection of a hypermodel, describing the second order statistics of h_t .

In this case study we consider autoregressive and possibly integrating models of order n_D , with equal dynamics for all channel taps, described by

$$h_t = \frac{1}{D(q^{-1})} \mathbf{I} e_t = \frac{1}{1 + d_1 q^{-1} + \dots + d_{n_D} q^{-n_D}} \mathbf{I} e_t \quad (4)$$

or

$$h_t + d_1 h_{t-1} + \dots + d_{n_D} h_{t-n_D} = e_t, \quad (5)$$

with real-valued scalar coefficients $\{d_i\}$. Here, q^{-1} denotes the backward shift operator and e_t is a white zero mean random vector sequence with covariance matrix \mathbf{R}_e . For example, when $n_D = 2$ the model is denoted AR₂, while for an AR₂I model, $n_D = 3$ and one pole is at $z = 1$.

When the Doppler speed is known, the model should be adjusted to the autocorrelation function (2). Perfect adjustment would require models of infinite degree, but good performance can be obtained with simple models. For AR₂ models, we use

$$D(q^{-1}) = 1 - 2\rho \cos \frac{\Omega_D^o}{\sqrt{2}} q^{-1} + \rho^2 q^{-2}$$

where Ω_D^o is the nominal maximal (normalized) Doppler frequency and $\rho = 0.999 - 0.1\Omega_D^o$, which works well for $\Omega_D^o < 0.1$.

For higher order AR models, we adjust $D(q^{-1})$ by considering row j of (5). Introduce the set of covariances

$$\{r_{\ell_i} \triangleq \mathbb{E} h_{j,t} h_{j,t-\ell_i}^*\}_{i=1}^N,$$

where $h_{j,t}$ denotes element (tap) j of h_t and where ℓ_i are integers such that $0 < \ell_1 < \dots < \ell_N$. Multiplying row j of (5) by $h_{j,t-\ell_i}^*$ and taking the expectation gives the set of equations

$$r_{\ell_i} + d_1 r_{\ell_i-1} + \dots + d_{n_D} r_{\ell_i-n_D} = 0; \quad i = 1, 2, \dots, N. \quad (6)$$

Assuming Jakes model, we replace r_{ℓ_i-w} by $J_0(\Omega_D(\ell_i-w))$ for a known or estimated Ω_D , and solve the possibly over-determined system of equations (6) by the least squares method [10].

When (4), \mathbf{R}_e , \mathbf{R} and σ_v are given, we can optimize a WLMS-algorithm for tracking h_t in (1):

$$\varepsilon_t = y_t - \varphi_t^* \hat{h}_{t|t-1} \quad (7)$$

$$\hat{h}_{t|t} = \hat{h}_{t|t-1} + \mu \mathbf{R}^{-1} \varphi_t \varepsilon_t \quad (8)$$

$$\hat{h}_{t+k|t} = \frac{Q_k(q^{-1})}{Q_0(q^{-1})} \mathbf{I} \hat{h}_{t|t}. \quad (9)$$

Here, $\hat{h}_{t+k|t}$ is an estimate of h_{t+k} at time t , μ is a scalar gain and $Q_k(q^{-1})/Q_0(q^{-1})\mathbf{I}$ in (9) is the coefficient smoothing-prediction filter. An equivalent implementation can be expressed in terms of the *learning*

filter $\mathcal{L}_k(q^{-1})$ [1, 9]:

$$\begin{aligned} f_t &= \mathbf{R} \hat{h}_{t|t-1} + \varphi_t \varepsilon_t \\ \hat{h}_{t+k|t} &= \mathcal{L}_k(q^{-1}) f_t = \frac{Q_k(q^{-1})}{\beta(q^{-1})} \mathbf{R}^{-1} f_t, \end{aligned} \quad (10)$$

where

$$\beta(q^{-1}) = D(q^{-1}) + q^{-1} Q_1(q^{-1}). \quad (11)$$

The polynomials $Q_k(q^{-1})$ depend on the selected hypermodel and on the SNR, and are calculated via Theorem 1 in [1] or [9] to minimize

$$E|\tilde{h}_{t+k|t}|^2 = \text{tr} E \tilde{h}_{t+k|t} \tilde{h}_{t+k|t}^* \triangleq \text{tr} \mathbf{P}_k \quad (12)$$

where $\tilde{h}_{t+k|t} = h_{t+k} - \hat{h}_{t+k|t}$.

IV. Tools for Performance Analysis

For one- or two-tap fading channels and symbol alphabets $\{u_t\}$ with constant modulus, there exists an exact expression for the MSE performance (12), for a given algorithm in a given fading environment [7, 8]. This expression is valid for arbitrary fast fading rates.

Lemma 1. Consider the channel model (1), with $M < 3$. Assume h_t , φ_t^* and v_t to be mutually independent and stationary. The spectrum $\phi_h(\Omega)$ of h_t is described by (3), and the zero mean noise v_t has variance σ_v^2 . Let the zero mean symbols u_t be white, with constant modulus and variance σ_u^2 ($\mathbf{R} = \sigma_u^2 \mathbf{I}$). Assume $(M-1)\Sigma_1 < 1$, where Σ_1 is defined in (14) below. If an estimator for h_{t+k} with the structure (10) or (7)–(9) is used, then the steady-state mean square estimation error (12) is given by

$$\text{tr} \mathbf{P}_k = \frac{\Gamma_k + M 10^{-\frac{\text{SNR}}{10}} \Sigma_k + (M-1)G_k}{1 - (M-1)\Sigma_1} \text{tr} \mathbf{R}_h \quad (13)$$

where

$$\Sigma_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \frac{Q_k(e^{j\Omega})}{\beta(e^{j\Omega})} \right|^2 d\Omega \quad (14)$$

$$\Gamma_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| \frac{\beta(e^{j\Omega}) - e^{j\Omega k} Q_k(e^{j\Omega})}{\beta(e^{j\Omega})} \right|^2 \frac{\text{tr} \phi_h}{\text{tr} \mathbf{R}_h} d\Omega \quad (15)$$

$$G_k = \Gamma_1 \Sigma_k - \Gamma_k \Sigma_1 \quad (16)$$

and

$$\text{SNR} \triangleq 10 \log \frac{\sigma_u^2}{\sigma_v^2} \mathbb{E} |h_t|^2 \quad (\text{dB}) \quad (17)$$

□

Proof: Given in [7], [8].

Above, $(M-1)\Sigma_1 < 1$ is a condition for convergence in MSE. This condition will always be fulfilled for flat fading channels. Note also that the term G_k vanishes for $k = 1$. All preconditions for Lemma 1 are fulfilled in the IS-136 TDMA system: The symbol sequence is white and circular with constant modulus. The delay spread is not larger than one symbol

interval T , so channel models with $M \leq 2$ are appropriate. The noise v_t represents mainly co-channel interference and can be assumed independent of both u_t and h_t .

Lemma 1 can be generalized to fading statistics other than the Jakes' Rayleigh fading model, by modifying the fading spectrum $\phi_h(\Omega)$ used in (15).

It is of interest to know to what extent improved linear regression modelling can improve the end result for which it is intended. Filtering or detection performance is essentially determined by the ambient SNR. With Lemma 1, the variance of the "tracking noise" $\sum_{i=0}^{M-1} \tilde{h}_{i,t} u_{t-i}$ at the channel model output, caused by non-perfect tracking, can be calculated and compared to the variance of the noise v_t . As a rough but useful performance indicator, we define the relative noise level

$$V \triangleq 10 \log \left(\frac{\sigma_u^2 \text{tr} \mathbf{P}_k + \sigma_v^2}{\sigma_v^2} \right) \quad (\text{dB}) \quad (18)$$

where the numerator describes the total tracking plus noise variance, if \tilde{h} , u and v are mutually uncorrelated.

When V is above 3dB, the tracking noise dominates over the output noise v_t . It is then worthwhile to consider a superior adaptation law based on, for example, a higher order hypermodel. If V is below 1dB, then the noise v_t dominates, so even the total elimination of any remaining tracking error would result in marginal improvements of the performance of a filter or detector based on the estimated model.

V. MSE Performance

The theoretical MSE according to Lemma 1 can be used, for example, to compare the MSE performance of WLMS algorithms based on hypermodels (4) of different complexity. This is done in Table 1 and the lower part of Figure 1 for two-tap Rayleigh fading channels with Jakes' statistics. We evaluate the use of a random walk (RW) model $h_t = h_{t-1} + e_t$ for which WLMS reduces to an LMS adaptation law. It is compared to the use of second and fourth order autoregressive models (AR₂ and AR₄), adjusted to the fading statistics. The transmitted QPSK symbols u_t are here assumed to be known. Results are given for maximal normalized Doppler frequencies Ω_D between 0.02 and 0.06, corresponding to 45 km/h and 137 km/h, respectively, at 1900 MHz.

It can be seen that the use of a higher model order improves the performance. At 15dB for example, a WLMS tracker based on AR₄ modeling provides a lower MSE at 137km/h than LMS tracking at 45km/h (Table 1). In terms of the effect of the noise level on the tracking MSE, more than 10 dB can be gained at both $\Omega_D = 0.02$ and $\Omega_D = 0.06$ by using an AR₄ model instead of a random walk model (Figure 1).

The top diagrams in Figure 1 display the relative tracking noise level (18), under the assumption $\sigma_u^2 = 1$, $E|h_{0,t}|^2 = E|h_{1,t}|^2 = 1$ and $\text{SNR} = 10 \log(2/\sigma_v^2)$.

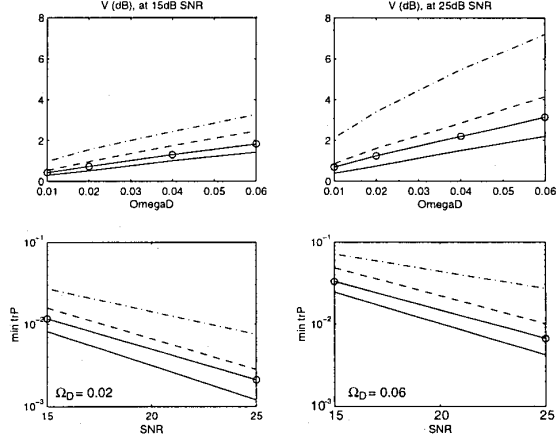


Figure 1: Optimized tracking error $E \|\tilde{h}_{t+1|t}\|_2^2 = \text{tr} \mathbf{P}_1$ (lower part) and relative tracking noise level V (dB) (upper part) in Section V for WLMS algorithms based on RW modelling (dashed-dotted), integrated random walks (dashed) AR₂ (circle) and AR₄ (solid). All AR models are matched to the true normalized Doppler frequency Ω_D .

Table 1: The attainable tracking error $\text{tr} \mathbf{P}_1$ for WLMS algorithms based on different hypermodels.

Ω_D	0.02		0.06		
	SNR	15 dB	25 dB	15 dB	25 dB
RW (LMS)		0.0271	0.0075	0.0711	0.0269
AR ₂		0.0117	0.0021	0.0329	0.0067
AR ₄		0.0082	0.0012	0.0245	0.0042

For LMS tracking (WLMS based on random walks), the tracking error is in many of the considered cases so large that it dominates the total noise ($V > 3\text{dB}$).

The performance and robustness of incorrectly tuned algorithms, computed by Lemma 1, is investigated in Figure 2. The algorithms were matched to a maximum Doppler frequency of 140Hz ($\Omega_D = 0.035$) and SNR 15 dB. The results indicate that the AR₂I hypermodel provides superior robustness if we *underestimate* the SNR and *overestimate* the Doppler frequency.

The use of predicted channel estimates has also been investigated and was shown to improve the tracking performance significantly [10].

VI. Simulation Study

We investigate the bit error rate performance of adaptive decision-directed Viterbi receivers in combination with LMS, AR₂ and AR₂I-based trackers. The best performance is obtained with a decision delay 3 in the detector. Due to an additional feedback delay, $k = 4$ step prediction of the channel is then required. (For LMS, $k = 3$ gives the best performance.)

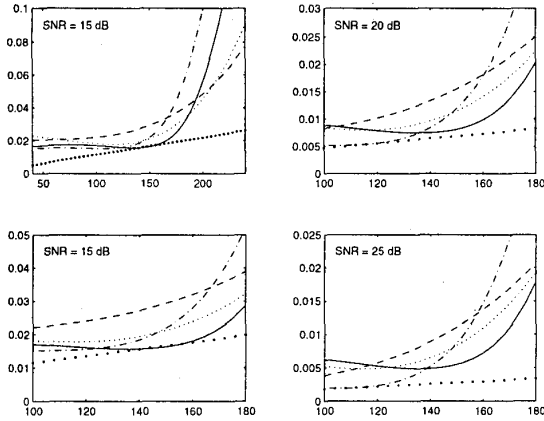


Figure 2: MSE performance $\text{tr} \mathbf{P}_1$ as a function of f_D for different choices of WLMS algorithms *matched to 140 Hz and 15 dB*. The algorithms are based on integrated random walk (dashed), AR₂ (dotted), AR₂I (dash-dotted) and AR₃ (solid) hypermodels. Compare to a fully matched AR₃ design (bulleted). The lower left-hand plot expands the upper left-hand diagram.

VI.A Specifications

We focus on a set-up suitable for the D-AMPS 1900 standard IS-136 with the following conditions.

- *Slot structure:* As in the forward link of IS-136 with $N = 162$ differential QPSK-modulated symbols, including 14 leading training symbols.¹
- *Channel properties:* A two tap Rayleigh fading symbol-spaced baseband channel model with independently fading taps² is simulated:

$$y_t = h_{0,t}u_t + h_{1,t}u_{t-1} + v_t ; \mathbf{R} = \mathbf{I} \quad (19)$$

with \mathbf{R}_e diagonal and $E|h_{0,t}|^2 = E|h_{1,t}|^2 = 1$. The taps $h_{i,t}$ are generated according to [3], using 12 offset oscillators with uniformly distributed $([0, 2\pi])$ phases. Hence, the level crossing statistics are close to classical Rayleigh fading. All estimators are initialized from least squares estimates of the channel taps in the form of robustified linear trends, based on the initial training sequence. We also study the flat fading case.

- *Disturbances:* The scenario is interference-limited with burst-synchronized interferers propagating via the same type of fading channel as the signal. The color of the interference is not estimated. (In a noise-limited scenario with Gaussian noise, the BER performance improves.)

¹A known CDVCC sequence of six differential symbols is placed after 85 symbols of the slot. They are here not used to improve the tracking performance, since this would complicate the performance evaluation.

²The more realistic case of correlated taps would result in higher bit error rates due to partial loss of diversity, but will otherwise not provide any new fundamental problems for the tracking.

- *Idealized simulation conditions:* We have compared decision directed adaptation to the use of correct symbols u_t as regressors. To quantify the loss of performance due to imperfect initialization, we also compare to initialization with known channel taps.

VI.B BER Performance for Two-tap Channels

Channels with two symbol-spaced taps of equal magnitude are simulated. The taps are independently Rayleigh fading, as described by Jakes' model, with normalized Doppler frequency $\Omega_D = 0.04$ (90 km/h at 1900 MHz). WLMS tracking algorithms based on random walk (LMS), AR₂ and AR₂I fading models are evaluated in combination with a Viterbi algorithm.

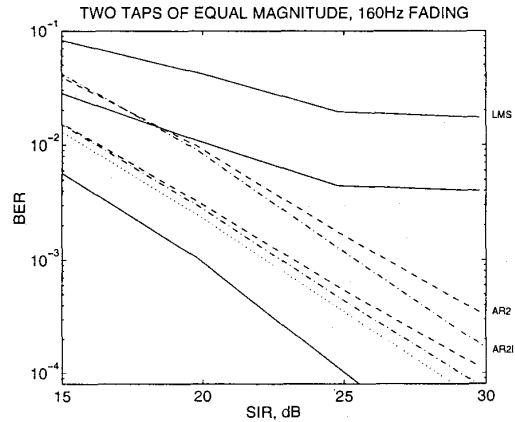


Figure 3: The Bit error rate as a function of the signal-to-interference ratio for the adaptive Viterbi equalizer with $k = 4$ ($k = 3$ for LMS). The BER with correct channel (lower solid) is compared to WLMS tracking with AR₂I modelling with true u_t as regressors (lower dash-dotted) and estimated regressors (upper dash-dotted) and to WLMS tracking with AR₂ modelling with true u_t as regressors (lower dashed) and estimated regressors (upper dashed). Compare to LMS with optimized step length and true u_t as regressors (middle solid) and with estimated regressors (upper solid). Also shown is AR₂I tracking using true u_t and correct initialization (dotted). 10000 slots are considered for each simulation case.

Figure 3 presents the uncoded bit error rate when the correct Ω_D and signal-to-interference ratios (SIRs) are used in the design.

Comparing the dotted to the lower dash-dotted curve in Figure 3 we see that not much performance is lost due to imperfect initialization. (If the algorithms were initialized with levels instead of linear trends, the performance would deteriorate further by 1-2dB.)

Decision-directed adaptation results in a performance loss due to nonlinear feedback effects. It is approximately 3dB for WLMS based on AR₂ and AR₂I models in Figure 3.

In Figure 3, WLMS based on AR₂I models show the best performance, but the performance of AR₂-based trackers is rather close. LMS tracking will in

this case be completely inadequate, partly due to its inappropriate structure and not least due to its inability to predict the channels; With a random walk model, $\hat{h}_{t+k|t} = \hat{h}_{t|t}$. This results in a significant lag error, which will not vanish at low disturbance levels. Hence, the error floor at 1.7% BER.

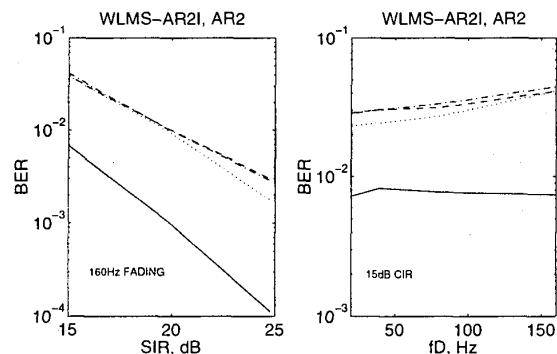


Figure 4: The BER as function of the SIR at 160Hz (left figure) and as a function of the Doppler frequency at 15dB SIR (right figure) for an adaptive Viterbi detector with $k=4$ step prediction. Performance of AR₂I-based (dash-dotted) and AR₂-based (dashed) WLMS channel estimators, designed for $SIR=15dB$ and $f_D = 160Hz$. Compare to the performance of AR₂-based WLMS designed for the true SIR and f_D (dotted) and to the performance for a known channel (solid).

To test our conclusions from Section V, we have designed AR₂ and AR₂I-based WLMS algorithms for $f_D = 160Hz$ and $SIR=15dB$ and evaluated their performance at other operating points. The results, presented in Figure 4, confirm that one single fixed adaptive filter, designed at the high end of the uncertainty interval of the Doppler frequency and the low end of the SIR range can indeed be used over the whole parameter range. If the operating area is bounded by $SIR=[15,25]dB$ and $f_D = [0, 160]Hz$, then this filter does in fact constitute a minimax robust design, since the so-called saddle-point condition [5] is fulfilled: The resulting performance attains its worst value at the nominal (worst-case) design point. In the most critical regions, with low SIR and/or high Doppler frequency, the performance for an AR₂I-based design is about the same as for an AR₂-based design.

VI.C BER Performance for Flat fading

In the flat fading case, with $h_{1,t} = 0$, not much can be gained by improving the tracking, see Table 2. An exception is at high SNR, where for true regressors a significantly lower BER is attained for AR₂ or AR₂I-based designs as compared to LMS. This can be predicted by the values of V from (18) in the right-hand part of Table 2. For flat fading channels, all the algorithms provide about the same performance. The detector becomes trivially simple, so no channel prediction beyond $k = 1$ is required.

Table 2: Flat fading at $\Omega_D = 0.04$, $E|h_t^0|^2 = 1$, $E|h_t^1|^2 = 0$. In row 4 to 6, a true symbol is used as regressor. The relative noise level (V) is obtained with $k = 1$.

SNR (dB):	BER (%)			μ and V	
	15	20	25	15	25
ESTIMATED REGRESSORS:					
LMS	2.8	1.00	.36	.39	.73
WLMS AR2	2.7	0.95	.32	.17	.25
WLMS AR2I	2.7	0.96	.33	.14	.19
TRUE REGRESSORS:					
LMS	3.1	1.24	.52	1.5	3.2
WLMS AR2	3.2	1.12	.37	0.7	1.2
WLMS AR2I	2.5	0.88	.31	0.6	1.0
Known channel	2.1	0.69	.23	0	0

References

- [1] A. Ahlén, L. Lindbom and M. Sternad, "Channel tracking with WLMS algorithms: High performance at LMS computational load," *VTC2000-Spring*, Tokyo, May 15-18 2000.
- [2] G. E. Bottomley and K. Molnar, "Adaptive channel estimation for multichannel MLSE receivers", *IEEE Communications Letters*, vol. 3, pp.40-42, Jan. 1999.
- [3] W. C. Jakes, "Multipath Interference". In W. C. Jakes ed. *Microwave Mobile Communications*. Wiley, New York, NY, 1974. Reissued by IEEE, Piscataway NJ.
- [4] K. Jamal, G. Brismark and B. Gudmundson, "Adaptive MLSE performance on the D-AMPS 1900 channel," *IEEE Trans. on Vehic. Tech.*, vol. 46, pp. 634-641, 1997.
- [5] S. A. Kassam and V. Poor, "Robust techniques for signal processing: a survey," *Proc. of the IEEE*, vol. 73, pp. 433-481, 1985.
- [6] L. Lindbom, "Simplified Kalman estimation of fading mobile radio channels," *IEEE ICASSP 1993*, Minneapolis, MN, April 27-30, vol. 3, pp. 352-355.
- [7] L. Lindbom, *A Wiener filtering approach to the design of tracking algorithms: with application in mobile radio communications*. PhD Thesis, Dept. of Technology, Uppsala University, Sweden, 1995. www.signal.uu.se/Publications/abstracts/a951.html
- [8] A. Ahlén, L. Lindbom and M. Sternad, "Tracking of time-varying systems, Part II: Analysis of stability and performance" Submitted. www.signal.uu.se/Publications/abstracts/r002.html
- [9] L. Lindbom, M. Sternad and A. Ahlén, "Tracking of time-varying mobile radio channels. Part I: The Wiener LMS algorithm," Submitted. www.signal.uu.se/Publications/abstracts/r003.html
- [10] L. Lindbom, A. Ahlén, M. Sternad and M. Falkenström, "Tracking of time-varying mobile radio channels. Part II: A case study on D-AMPS 1900 channels," Submitted. www.signal.uu.se/Publications/abstracts/r004.html
- [11] K. J. Molnar and G. E. Bottomley, "Adaptive array processing MLSE receivers for TDMA digital cellular PCS communications." *IEEE J. Selected Areas in Commun.*, vol. 16, pp. 1340-1351, Oct. 1998.