Analysis and Identification of Transmitter Non-linearities

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Abstract

In this article the effects of non-linearities and other impairments in a direct up-conversion transmitter is investigated. A method to find the different parameters of the non-linearities and of the impairments without doing internal measurements is developed and simulated and its robustness against divergence from ideal conditions and against noise is investigated.

By using simple non-linear models for the non-linearities at baseband and for the power amplifier, PA, a total mathematical model of the transmitter is obtained. From the model it can be seen that both the amplitude and the phase distortion of the transmitter are dependent on both the phase and the envelope of the input signal. The parameters of this model is then identified by studying the amplitude and phase distortion at the output. The impact of noise and of modelling errors are studied by simulations and it is found that the method is robust against modelling errors and that the sensitivity to noise is strongly dependent on the strength of the non-linearity. If the identified parameters are used to distort a two-tone input signal, the error in third order intermodulation products are less than 0.5 dB for both the baseband alone, for the PA alone and for the system as a whole, except for the case of low signal-to-noise ratio.

I. Introduction

A transmitter suffers from a number of imperfections, the most notably being the non-linear PA. However, in a direct up-conversion transmitter, there are other imperfections which can be of interest. There are non-linear mixers, amplifiers and digital-to-analogue converters, DACs, at baseband, there can be a DC offset, the two signal paths do not necessarily have equal gain and the 90 degree phase shifter may have an error. In this article, those impairments and their effect on the RF output will be discussed and a method to separate and identify the different impairments will be developed. In **II** the output of the transmitter is derived, in **III**, the parameters of the impairments are found, in **IV**, the method is simulated and in **V** conclusions are drawn.

II. Deriving the Output of the Transmitter

If the signal is represented by its equivalent lowpass form, the output of the modulator, i.e. the input to the PA, is

$$s_{l,\mathrm{mod}}(t) = g_I(x(t)) - g_Q(y(t))\sin\theta_0 + jg_Q(y(t))\cos\theta_0 \qquad (1)$$

where $g_I(\bullet)$ and $g_Q(\bullet)$ represents the non-linearities of the in-phase and quadrature-phase signal paths at baseband respectively. θ_0 is the phase error in the 90 degree phase shifter and $x(t)$ and $y(t)$ are the in-phase and quadrature-phase signals respectively. If the non-linear PA envelope model given in [1] is used, the total output becomes

$$s_l(t) = h(a(t)) = g(a(t))e^{j(j(t)+f(a(t)))} \qquad (2)$$

where $g(\bullet)$ and $f(\bullet)$ are referred to as the AM/AM and AM/PM distortion respectively, $j(t)$ and $a(t)$ are the phase and the envelope of $s_{l,mod}(t)$ given by

$$a(t) = \sqrt{(g_I(x(t)) - g_Q(y(t))\sin q_0)^2 + g_Q^2(y(t))\cos^2 q_0} \qquad (3a)$$

$$j(t) = \arctan\left(\frac{g_Q(y(t))\cos q}{g_I(x(t)) - g_Q(y(t))\sin q_0}\right) \qquad (3b)$$

The total output given by (2) can now be rewritten as

$$s_l(t) = \hat{g}(r(t), f(t))e^{j(f(t)+e(r(t),f(t)))} \qquad (4)$$

where $r(t)$ and $f(t)$ are the ideal envelope and phase, $\varepsilon(r(t), f(t))$ is the total phase distortion and $\hat{g}(\bullet)$ is the AM/AM distortion as a function of $r(t)$ and $f(t)$.

III. Identifying the non-linearities

A. The Identification

The total phase error, $\varepsilon(r(t), f(t))$, at sampling instant n, $n = 0, 1, 2, \ldots, N$, where N is the number of samples, can now be written as, if n is omitted

$$e(r, f) = j - f + f(a) =$$
$$= \arctan\left(\frac{g_Q(y)\cos q_0}{g_I(x) - g_Q(y)\sin q_0}\right) - \arctan\left(\frac{y}{x}\right) + f(a) \approx$$

$$\approx \{x \neq 0\} \approx \frac{xg_Q(y)\cos q_0 - y(g_I(x) - g_Q(y)\sin q_0)}{x(g_I(x) - g_Q(y)\sin q_0) + yg_Q(y)\cos q_0} +$$
$$+ f((g_I(x) - g_Q(y)\sin q_0)^2 + g_Q^2(y)\cos^2 q_0)^{1/2} \Leftrightarrow$$

$$\Leftrightarrow xg_Q(y)\cos q_0 - y(g_I(x) - g_Q(y)\sin q_0) +$$
$$+ f((g_I(x) - g_Q(y)\sin q_0)^2 + g_Q^2(y)\cos^2 q_0)^{1/2} \cdot \qquad (5)$$
$$\cdot (x(g_I(x) - g_Q(y)\sin q_0) + yg_Q(y)\cos q_0) \approx$$
$$\approx (x(g_I(x) - g_Q(y)\sin q_0) + yg_Q(y)\cos q_0)e(r, f)$$

It can easily be guaranteed that $x(n) \neq 0$ by simply not using these samples in the identification. The non-linearities $g_I(\bullet)$, $g_Q(\bullet)$ and $f(\bullet)$, are then approximated with functions, here polynomials are used.

The envelope, $a(n)$, is then calculated using (3a). As for the other non-linearities, $g(\bullet)$ is approximated with a polynomial. With the knowledge of $a(n)$, the coefficients of $g(\bullet)$ is obtained by solving the following system where the linear gain has been normalised to one.

$$|s_l(n)| = g(a(n)) = a(n) + g_3a^3(n) + g_5a^5(n) + \ldots \qquad (6)$$

where $n = 1, 2, \ldots, N$ and N is the number of samples.

B. Error Sources

In a noise free environment, the only errors in the identification comes from the approximations done when the non-linearities are approximated with functions and when inserting these approximations in (5). These errors can be made smaller by using more terms in the approximations.

If the real system is of higher order than the model, it is desirable that the identification should still give a fairly good approximation. It is first noted that if the polynomial used to approximate $g(\bullet)$ is of too low order, it will not affect the phase, which implies that the identification of $g_I(\bullet)$, $g_Q(\bullet)$ and $f(\bullet)$ is unaffected and that the coefficients of $g(\bullet)$ will be the best possible in a least-mean-square sense. If the model order is lower than the real world system order for any of $g_I(\bullet)$, $g_Q(\bullet)$ or $f(\bullet)$, the entire identification will be affected.

The identification will be affected by noise, to what extent is dependent on the strength of the non-linearity, the number of terms that are to be identified and the signal-to-noise ratio. Generally, high signal-to-noise ratio and strong non-linearities, will give the best identification performance.

IV. Simulations

A. The Identification

To evaluate the identification method and to study the effect of the imperfections mentioned earlier, a certain system given in (7), where *n* is implicit, was simulated. Here the input signal used consisted of two tones, both with amplitude 0.5 and zero phase. The phase and envelope errors for this system is shown in fig. 1 and 2. From fig. 1 and 2 it is clear that both the envelope and the phase errors have parts which are not functions of the envelope only.

$$g_I(x) = 0.001 + x + 0.002x^2 - 0.003x^3 + \\ + 0.0001x^4 - 0.0002x^5 \tag{7a}$$

$$g_Q(y) = -0.0007 + 1.01y - 0.001y^2 - 0.004y^3 + \\ + 0.0002y^4 - 0.0003y^5 \tag{7b}$$

$$f(a) = 0.01a^2 - 0.003a^4 \tag{7c}$$

$$g(a) = a - 0.01a^3 - 0.0008a^5$$

(7d)

$$\theta_0 = 0.5^o \tag{7e}$$

B. The Effects of the Approximations

If the system given by (7) is identified, without any noise added, the result is

$$g_I(x) = 0.00100 + x + 0.00200x^2 - 0.00281x^3 + \\ + 0.00006x^4 - 0.00019x^5 \tag{8a}$$

$$g_Q(y) = -0.00070 + 1.0100y - 0.00103y^2 - 0.00384y^3 + \\ + 0.00017y^4 - 0.00037y^5 \tag{8b}$$

$$f(a) = 0.00993a^2 - 0.00307a^4 \tag{8c}$$

$$g(a) = a - 0.01027a^3 - 0.00066a^5 \tag{8d}$$

$$\theta_0 = .5005^o \tag{8e}$$

The spectral effects of the errors in (8) at the output of the modulator is about 0.4 dB for the third order intermodulation, *IM3*, and less than 0.1 dB for the image, the carrier leakage, *CL*, and the fifth order intermodulation, *IM5*. At the output of the transmitter, the error is less than 0.1 dB for all of the largest spectral components. If the coefficients of *g*(•) and *f*(•) are used to model the PA only, the error is less than 0.1 dB for the *IM3* and less than 0.2 dB for the *IM5*.

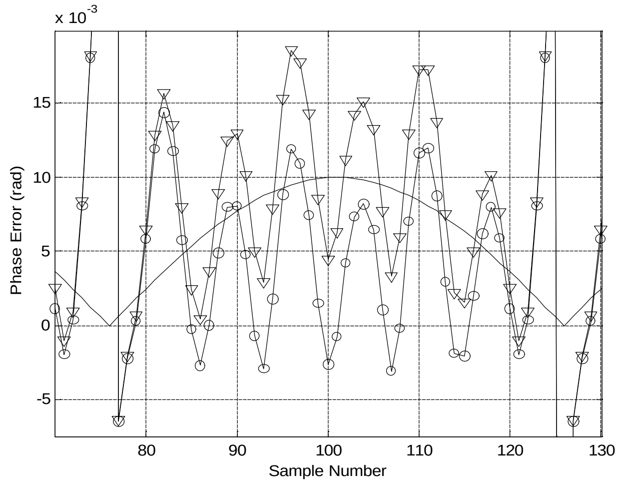

Fig. 1. The phase error at the output of the modulator, o, and at the output of the PA, triangle. The solid line is the envelope of the signal divided by 100.

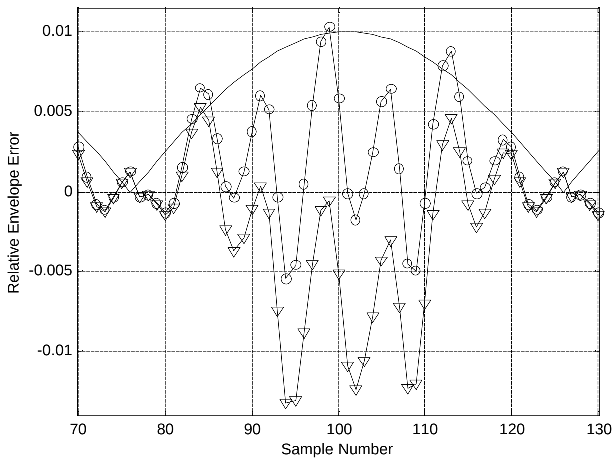

Fig. 2. The envelope error at the output of the modulator, o, and at the output of the PA, triangle. The solid line is the envelope of the signal divided by 100.

C. The Effects of Higher-Order Non-Linear Systems

Suppose that the real system given by (7) is extended to seventh order, the extra terms are g_{i6} = 0.0001, g_{i7} = -

0.0001, g_{q6} = 0.0001, g_{q7} = -0.0002, g_7 = -0.0003 and f_6 = 0.001, the identified system then becomes

$$g_I(x) = 0.00104 + x + 0.00197x^2 - 0.00319x^3 + \\ + 0.00026x^4 - 0.00036x^5 \quad (9a)$$

$$g_Q(y) = -0.00072 + 1.0100y - 0.00108y^2 - 0.00448y^3 + \\ + 0.00039y^4 - 0.00032y^5 \quad (9b)$$

$$f(a) = .00946a^2 - .00132a^4 \quad (9c)$$

$$g(a) = a - .00927a^3 - .00155a^5 \quad (9d)$$

$$\theta_0 = 0.5132^o \quad (9e)$$

The values of the coefficients have changed from (8). However, this is as it should be since the coefficients in (9) should ideally give the best possible fifth order approximation to the seventh order system. What is interesting is the spectral performance. The result is that for the total output, the error is less than 0.5 dB for the *CL* and less than 0.1 dB for the image and the *IM3*. For the modulator the error is less than 0.5 dB for the *CL*, less than 1 dB for the *IM3* and less than 0.1 dB for the image. For the PA the error is less than 0.1 dB for the *IM3* and less than 1 dB for the *IM5*.

D. The Effects of Noise

The effects of noise are, as mentioned earlier, dependent on the magnitude of the imperfections and on the order of the model. For example, let the signal-to-noise ratio be 74 dB and let the system be given by (12). If the spectra generated by (12) and the spectra obtained by distorting the input signal by the identified coefficients are compared, it is found that the error is less than 1 dB for the image, the CL, the *IM3* of the PA, the *IM2* of the baseband, about 3 dB for the *IM5* of both the PA and the baseband and for the *IM3* of the baseband and that it is more than 10 dB for the *IM4* of the baseband. If instead the signal-to-noise ratio is 94 dB, then the error is less than 0.3 dB for all spectral components except the *IM4*, for which it is approximately 2 dB.

VI. Conclusions

A method to identify and separate the non-linearities at baseband and at RF in a direct up-conversion transmitter without doing internal measurements has been developed. The method has been verified by simulations and it has been found that if estimates of the parameters are used to distort a two-tone input signal, the errors of the different spectral components are normally only a few tenths of a dB, which must be considered good.

The simulations done with a higher-order non-linear system and with noise added, suggests that the method is robust against distortion from the ideal case and against noise, even though a high signal-to-noise ratio will be needed for weak non-linearities.

VII. Acknowledgement

The authors want to thank Ericsson Radio System AB in Gävle for their support and especially Mr. Olav Andersen, Mr. Peder Malmlöf, Mr. Anders Svensson and Mr. Niclas Keskitalo for support and fruitful discussions.

VIII. References

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